Math 102

Truth Values for Statements with More Than One Logical Connective

Now that we know the basic truth tables for each of the 5 logical connectives, we need to learn how to piece these together to make truth tables the will allow us to understand the final truth values for more complicated compound statements.

The goal is to be able to find the truth value of the compound expression for any possible assignment to truth values to the original simple statements that the compound statement is built from. We will do this by using the following procedure:

- First, look at the compound statement and count the number of simple statements found in the expression. This tells us how many different truth value assignments the compound statement has, so we know how many rows will be in our truth table. Add a column to the truth table for each simple statement and then add rows for each possible truth value.
- Next, carefully examine the compound statement, paying attention to the order of operations, and number each logical connective in the order that it is carried out.
- Then add a row for each logical connective. Fill in the row by looking at the truth value(s) from the column(s) the new logical connective is being applied to using the basic rule for that connective.
- Once this has been done for each logical connective, the final column in the truth table represents the truth values for the full statement.

Note: You will be expected to use the "standard order" when writing out the truth value assignments in the truth table for a compound statement.

Examples: Build the truth table for each of the following compound statements.

- $\sim p \wedge q$

p	q	$\sim p$	$\sim p \wedge q$
T	T		
T	F		
F	T		
F	F		

- $p \rightarrow \sim q$

p	q	$\sim q$	$p \longleftrightarrow \sim q$
T	T		
T	F		
F	T		
F	F		

- $\sim (p \vee q)$

p	q	$p \vee q$	$\sim (p \vee q)$
T	T		
T	F		
F	T		
F	F		

- $\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T			
T	F			
F	T			
F	F			

Notes:

- Notice that the final column if the truth tables for $\sim (p \vee q)$ and $\sim p \wedge \sim q$ are identical. This means that for any assignment of truth values to the simple statements p and q, the final truth values of these two compound statements end up begin the same.
- We say that two symbolic logical statements are **logically equivalent** if they contain the same variables and have truth tables with identical final columns. The idea here is that although the two statements may look different, they have the same logical meaning since their truth values are always identical.
- The truth tables we built above show that the statements $\sim (p \vee q)$ and $\sim p \wedge \sim q$ are logically equivalent. In fact, the equivalence $\sim (p \vee q) \equiv \sim p \wedge \sim q$ is one of DeMorgan's Laws.
- As an exercise, use truth tables to verify the other version of DeMorgan's Laws: $\sim (p \wedge q) \equiv \sim p \vee \sim q$.
- A logical statement that is always true is called a **tautology**. The following truth table shows that the statement $p \vee \sim p$ is a tautology.

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Some Harder Examples:

- Build the truth table for the statement $p \wedge (q \vee r)$.

  Notice that there are *three* distinct variables in this symbolic statement. Consequently, there are more possible truth assignments. Since there are 2 possible truth values for each variable and there are 3 variables, there are $2 \times 2 \times 2 = 8$ possible truth values.

  In general, a statements with k variables has 2^k possible truth values assignments. [1 variable – 2 truth values, 2 variables – 4 truth values, 3 variables – 8 truth values, 4 variables – 8 truth values, etc.]

  We can use a tree diagram to help us fill in all possible truth values, or we can just learn the pattern of the standard order for these truth values.

  We Follow the same procedure as before. We just have more rows to fill in. One could build a truth table for a logical expression with any number of variables and any number of logical connectives. [Although it could become tedious!]

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	
T	T	F	T	
T	F	T	T	
T	F	F	F	
F	T	T	T	
F	T	F	T	
F	F	T	T	
F	F	F	F	

- Build the truth table for the statement $p \longleftrightarrow (q \wedge r)$.

p	q	r	$q \wedge r$	$p \longleftrightarrow (q \wedge r)$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		