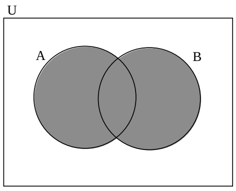
Math 102
Set Operations

The Universal Set: When we are working in a general context, the set of all objects currently under consideration is called the **universal set**. For example, if we were discussing grades of the previous exam, the universal set would be every student who took the exam. If we were talking about signing up for classes for the upcoming Spring semester, then the universal set would be all courses that will be offered during Spring semester 2011.

Set Operations: Once more than one set is in view, there are several operations that can be used to combine sets. We will look at four main set operations.

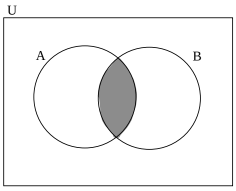
- The **union** of two sets A and B, denoted by $A \cup B$ is the set formed by combining the elements from the two sets to form a single set. Formally, we write this as $A \cup B = \{x : x \in A \text{ or } x \in B\}$

  The following Venn diagram illustrates the set operation $A \cup B$:

- The **intersection** of two sets A and B, denoted by $A \cap B$ is the set formed by taking all elements that are in *both* of the original sets. Formally, we write this as $A \cap B = \{x : x \in A \text{ and } x \in B\}$

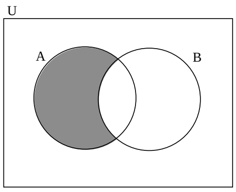
  The following diagram illustrates the set operation $A \cap B$:

- The **difference** of two sets A and B, denoted by $A - B$ is the set formed by starting with the elements that are in A and *removing* from A all of the elements it shared with B. Formally, we write this as $A - B = \{x : x \in A \text{ and } x \notin B\}$

  Warning: Order matters when finding set differences. In general $A - B \neq B - A$.

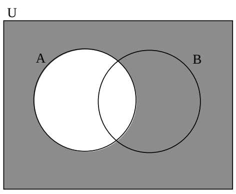
  The following diagram illustrates the set operation $A - B$:

- The **complement** of a set A relative to a universal set U, denoted by A' is the set formed by taking all of the elements in U but *not* in A. We think of A' as set set of elements outside A or as the "opposite" of the set A. Formally, we write this as $A' = \{x : x \in U \text{ and } x \notin A\}$

  Warning: We need to clearly understand what the universal set U is in order to find A'. If we were to change universal sets, we would have a different set complement.

  The following diagram illustrates the set operation A':

Examples: Lets practice carrying out set operations. We will begin by looking at some sets given in roster notation.

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5\}$, and $C = \{1, 3, 6, 7\}$.

$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$

$A \cap B = \{2, 4\}$

$A' = \{1, 3, 5, 7, 9\}$

$B' = \{6, 7, 8, 9, 10\}$

$A - B = \{6, 8, 10\}$

$B - A = \{1, 3, 5\}$

$(A \cup B)' = \{1, 2, 3, 4, 5, 6, 8, 10\}' = \{7, 9\}$

$A \cup (B \cap C) = \{2, 4, 6, 8, 10\} \cup \{1, 3\} = \{1, 2, 3, 4, 6, 8, 10\}$

$C - (A \cup B) = \{1, 3, 6, 7\} - \{1, 2, 3, 4, 5, 6, 8, 10\} = \{7\}$

Examples: We can also think about understanding the effect of set operations on sets that are given in set builder notation. Let $U = \{p : p$ is a person living in the U.S. $\}$, $A = \{p : p$ is a person who has brown hair $\}$, $B = \{p : p$ is a person who is over 6 feet tall $\}$.

$A \cap B = \{p : p$ is a person who has brown hair and who is over 6 feet tall $\}$

$A - B = \{p : p$ is a person who has brown hair but is *not* over 6 feet tall $\}$

$B - A = \{p : p$ is a person who is over 6 feet tall but who does *not* have brown hair $\}$

$(A \cup B)' = \{p : p$ is a person who does *not* have brown hair and who is *not* over 6 feet tall $\}$

Note: Set operations should remind you of the logical operations that we covered earlier this semester. Which logical connective corresponds to each of the set operations we are learning?

DeMorgan's Laws for Sets: Recall that DeMorgan's laws for logical connectives gave us the following:

$\sim (p \wedge q) \equiv \sim p \vee \sim q$ and $\sim (p \vee q) \equiv \sim p \wedge \sim q$.

If we rephrase these in terms of set operations, we have:

$(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$.

Exercise: Verify that DeMorgan's Laws for sets are true for the sets given in roster notation we used for examples at the top of this page.

Finding the Number of Elements in a Union: Given two sets A and B, the following formula can be used to directly compute the number of elements in the union of these two sets:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Why does this work? Recall that $n(A)$ is the number of elements in the set A and $n(B)$ is the number of elements in the set B.

If we take $n(A) + n(B)$, when adding the number of elements in these two sets, if there are any elements in **both** the these sets, we will have counted each of these elements *twice*. Because of this, in order to get a count of the elements in the union, we need to subtract off any element that was counted twice in order to get the true total.

Exercise: Verify that the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ is true for the sets $A = \{2, 4, 6, 8, 10\}$ and $B = \{1, 2, 4, 5\}$.