

### Definitions and Notation:

- A **set** is a well defined collection of objects. By “well defined”, we mean that it must be clear from the way the set is described whether any given object is in the set or not in the set.
- Intuitively, we sometimes think of a set as a “container” that “holds” certain objects.

### Examples:

The set of people in this room.

The set of courses offered at MSUM during Fall semester 2010.

The set of even numbers.

**Describing Sets:** There are 3 main ways of describing sets.

- **Verbal Descriptions:** a verbal description of a set uses an English sentence to state a rule that allows us to determine the class of objects being discussed and to determine for any particular object whether or not it is in the set.

The three sets in the examples above are all given using a verbal description.

- **Roster Notation:** In roster notation, we give a comprehensive listing of the elements in the set. The list begins and ends with curly braces. Elements on the list is separated by commas. We sometimes name the set using a capital letter.

### Examples:

$C = \{ \text{penny, nickel, dime, quarter, half-dollar, dollar} \}$

$V = \{ a, e, i, o, u \}$

$O = \{ 2, 4, 6, 8, 10, 12, 14, \dots \}$

$P = \{ 2, 3, 5, 7, 11, 13, 17, \dots \}$

**Note:** Since the last two examples are infinite sets, we cannot actually list *every* element in the set. For infinite sets, even though we are being somewhat imprecise, we will generally list enough elements to show the pattern, then use “...” to indicate that the set continues in a similar fashion.

- **Set-Builder Notation:** In set-builder notation, instead listing of the elements in the set, we define a *set variable*, followed by a verbal description of the category of elements and the rule used to determine which elements are in the set and which are not. The description still begins and ends with curly braces. The set variable and verbal description are separated either by a colon “:” or a vertical slash “|”. We still will sometimes name the set using a capital letter.

### Examples:

$C = \{ c : c \text{ is a type of coin in circulation as U.S. currency} \}$

$V = \{ v | v \text{ is a letter that always functions as a vowel} \}$

$O = \{ n | n \text{ is a positive even number} \}$

$P = \{ p : p \text{ is a prime number} \}$

**Note:** Whatever method we use to describe sets, it is important for us to make sure that the resulting set is well defined. That is, the rule or description for the set must make it clear precisely which objects are in the set and which are not.

**Examples:** Which of the following sets are well defined?

$$C = \{c : c \text{ is a car that was parked on campus at 8:00am this morning. } \}$$

$$T = \{t | t \text{ is a class that is difficult } \}$$

$$W = \{w | w \text{ is a three letter word. } \}$$

$$P = \{p : p \text{ is a person who is friendly. } \}$$

**Some “Famous” Sets:**

$$\mathbb{N} = \{x | x \text{ is a natural number } \} = \{1, 2, 3, 4, 5, \dots\}$$

$$\mathbb{W} = \{x | x \text{ is a whole number } \} = \{0, 1, 2, 3, 4, 5, \dots\}$$

$$\mathbb{Z} = \{x | x \text{ is an integer } \} = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

$$\mathbb{Q} = \{x | x \text{ is a rational number } \} = \{x | x \text{ can be written as a fraction of the form } \frac{a}{b}, a, b \text{ integers and } b \neq 0\}$$

$$\mathbb{R} = \{x | x \text{ is a real number } \}$$

$$\emptyset = \{ \}. \text{ We call this the } \mathbf{empty \ set}. \text{ It is a set with } \mathit{no \ elements}.$$

**Cardinality:** The **cardinality** of a set is the number of objects in the set. We use the notation  $n(A)$  to denote the cardinality of the set  $A$ .

**Examples:** Find the cardinality of each of the following sets.

$$A = \{1, 3, 5, 7, 9\} \qquad n(A) = \underline{\hspace{2cm}}$$

$$B = \{l | l \text{ is a letter in the word } \mathit{banana}\} \qquad n(B) = \underline{\hspace{2cm}}$$

$$C = \{n | n \text{ is a natural number greater than 5 and less than 20 } \} \qquad n(C) = \underline{\hspace{2cm}}$$

$$D = \emptyset \qquad n(D) = \underline{\hspace{2cm}}$$

$$E = \{\emptyset, \{0, 1\}\} \qquad n(E) = \underline{\hspace{2cm}}$$

$$F = \{x | x \text{ is a real number between 0 and 1}\} \qquad n(F) = \underline{\hspace{2cm}}$$

**Note:** If a set  $A$  does not have a finite number of elements, we say that  $n(A)$  is infinite.

**Elements or Sets and Subsets:** Consider the set  $A = \{1, 2, 5, 7, 8, 10\}$

**Notation:**

- Notice the the number 1 *is* an element of the set  $A$ . We denote this by writing  $1 \in A$ .
- Notice the the number 3 *is not* an element of the set  $A$ . We denote this by writing  $3 \notin A$ .
- A **subset** of the set  $A$  is a set  $B$  so that each element of  $B$  is also an element of the set  $A$ . In this case, we write  $B \subseteq A$ . For example, if  $B = \{2, 5, 7\}$ , then  $B \subseteq A$ .
- A **proper subset** of the set  $A$  is a set  $B$  so that each element of  $B$  is also an element of the set  $A$  **and**  $B \neq A$ . In this case, we write  $B \subset A$ .

**Examples:** Fill in the appropriate symbol for each of the following, given that  $A = \{1, 2, 5, 7, 8, 10\}$

1.  $5 \square A$

4.  $\{2, 7, 8\} \square A$

2.  $9 \square A$

5.  $\{7, 8, 9\} \square A$

3.  $\emptyset \square A$

6.  $A \square A$