Equality and Equivalence of Sets:

Definitions:

- Two sets are **equivalent** if they have the *same number of elements* [that is, if there is a 1-1 correspondence between the elements in the two sets].
- Two sets are **equal** if they have precisely the *same elements.*
- If two sets are equal then that are also equivalent. However, two equivalent set are not necessarily equal sets.

Examples:
Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{a, e, i, o, u\}$.

Then A and B are ______________.

Let $A = \{1, 2, 3\}$ and $B = \{1, 1, 2, 2, 3, 3\}$.

Then A and B are ______________.

Let $A = \{l \,|\, l$ is a letter in the word 'Mississippi' $\}$ and $B = \{l \,|\, l$ is a letter in the word 'bandana' $\}$.

Then A and B are ______________.

Let $A = \{l \,|\, l$ is a letter in the word 'banana' $\}$ and $B = \{l \,|\, l$ is a letter in the word 'bandana' $\}$.

Then A and B are ______________.

Let $A = \{n \,|\, n$ is a positive *even* integer $\}$ and $B = \{n \,|\, n$ is a positive *odd* integer $\}$.

Then A and B are ______________.

Finding the Subsets of a Given Set:

- Consider the set $\{a, b, c\}$. Our goal is to list **every** possible subset of this set.

Subsets of size 0:	$\emptyset$	[there is **1** subset of size 0]
Subsets of size 1:	$\{a\}$, $\{b\}$, $\{c\}$	[there are **3** subsets of size 1]
Subsets of size 2:	$\{a, b\}$, $\{a, c\}$, $\{b, c\}$	[there are **3** subsets of size 2]
Subsets of size 3:	$\{a, b, c\}$	[there is **1** subset of size 3] (although this subset is not a proper subset)

Total Number of Subsets: 8 [only 7 proper subsets]

- Consider the set $\{a, b, c, d\}$. Our goal is to list **every** possible subset of this set.

Subsets of size 0:	$\emptyset$	[there is **1** subset of size 0]
Subsets of size 1:	$\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$	[there are **4** subsets of size 1]
Subsets of size 2:	$\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$	[there are **6** subsets of size 2]
Subsets of size 3:	$\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$, $\{b, c, d\}$	[there are **4** subsets of size 3]
Subsets of size 4:	$\{a, b, c, d\}$	[there is **1** subset of size 4] (although this subset is not a proper subset)

Total Number of Subsets: 16 [only 15 proper subsets]

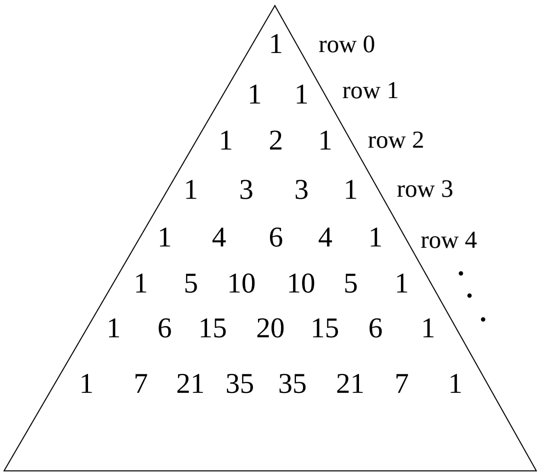
Claim: A set A with k elements (that is, $n(A) = k$) has exactly 2^k different subsets and $2^k - 1$ proper subsets.

Pascal's Triangle: As we look at larger and larger sets, it becomes difficult and tedious to list out all possible subsets of the set one at a time. A wonderful device that allows us to determine (for **any** finite set) the number of subsets it has of a specified size.

The device that we will use is called **Pascal's Triangle**. Here are the instructions to build each row of the triangle:

- Begin by placing a 1 by itself in the top row of the triangle.
- Next, build the second row by placing a 1 below and to the left of the 1 in the first row and then placing a 1 below and to the right of the 1 in the top row.
- Each successive row will have one more entry in it than the previous row.
- To build the next row, start and end with a 1s. Any entry between the first and last 1s is found by taking the sum of the entry above and to the left and the entry above and to the right of the new entry.

  See the diagram below to see the first few rows of Pascals's Triangle.



Note: Row 3 and Row 4 of Pascal's triangle are identical to the information we found in the examples we worked on the previous page.

Using Pascal's Triangle: Given a set, to use Pascal's Triangle to find the number of subsets of a certain size, we use the following procedure.

- Find the number of elements in the set you are given. The cardinality of the set is the same as the number of the row in Pascal's Triangle that we will reference. Looking at the second entry in this row is a nice way to double check that you are looking at the correct row since this entry gives the number of elements in that set type.
- Find the column within the correct row that lists the number of subsets of the the size we are looking for. To do this, we will count starting from zero up to the subset size that we want.

Caution: It is vital that we remember to count starting from 0 rather than 1 both when finding the row and the column in the triangle. Otherwise, we will be looking at the wrong entry and thus we will not get the correct numerical answer.

Examples: Use Pascal's Triangle to find each of the following.

- If $A = \{a, b, c, d, e\}$, find the number of subsets of A of size 3.
- If $A = \{a, b, c, d, e, f, g\}$, find the number of subsets of A of size 2.
- If $A = \{a, b, c, d, e, f, g, h\}$, find the number of subsets of A of size 4.