Math 102
Exam 1 **Name:**______________________________

1. (2 points each) Determine whether or not each of the following are statements:

   (a) O say does that star spangled banner yet wave o'er the land of the free and the home of the brave?

   Not a Statement.

   (b) Jack and Jill went up the hill to fetch a pail of water.

   Statement.

2. (3 points each)Given p : roses are red, q : violets are blue, and r : sugar is sweet, translate the following statements into symbolic form:

   (a) Roses are not red but sugar is sweet.

   $\sim p \wedge r$

   (b) If violets are blue or roses are red, then sugar is not sweet.

   $(q \vee p) \rightarrow \sim r$

3. (3 points each) Negate each of the following statements. Make sure to write your final negation in fully simplified English.

   (a) It rained every day last week.

   It did not rain at least one day last week.

   (b) Sometimes I am tired at the end of the workday.

   I am never tired at the end of the workday.

   (c) I will buy a new coat or I will buy a new hat.

   I will not buy a new coat and I will not buy a new hat.

4. (3 points each) Given p : I travel to Canada every year; q : I went fishing at a mountain lake; r : I have a hunting license; s : I got lost. Translate the following statements into words:

   (a) $(r \wedge \sim s) \rightarrow p$

   If I have a hunting license and I do not get lost, then I travel to Canada every year.

   (b) $\sim (p \wedge r)$

   It is not the case that I travel to Cananda every year and I have a hunting license.
   [or, using De Morgan's Law: I do not travel to Canada every year or I do not have a hunting license.]

5. (3 points each) For each of the following, decide whether the "or" used in the situation described is "inclusive" or "exclusive". Make sure to briefly explain your reasoning.

   (a) I will go for a jog after work or I will go shopping this evening.

   This is an example of an inclusive or. Notice that it is possible to do both activities.

   (b) I will dye my hair blue or I will leave it its natural color.

   This is an example of exclusive or. Notice that one cannot both dye one's hair blue and leave it its natural color.

6. (2 points each) Given the statements: p : I enjoy watching foreign films q : I go to the Fargo Theater.

   (a) Write the conditional statement relating p to q in words.

   If I enjoy watching foreign films than I go to the Fargo Theater.

   (b) Write the contrapositive in words.

   If I do not go to the Fargo Theater then I do not enjoy watching foreign films.

   (c) Write the inverse in words.

   If I do not enjoy watching foreign films then I do not go to the Fargo Theater.

7. (8 points) According to De Morgan's Law, $\sim (p \vee q)$ is logically equivalent to $\sim p \wedge \sim q$. Use truth tables to prove that these two statements are logically equivalent.

p	q	$p \vee q$	$\sim (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Since the last column in these two truth tables match, the statements are logically equivalent.

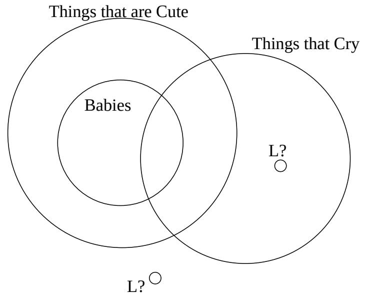
8. (8 points) Build a truth table for the statement: $(\sim p \to q) \vee r$

p	q	r	$\sim p$	$\sim p \to q$	$(\sim p \to q) \vee r$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	F	F

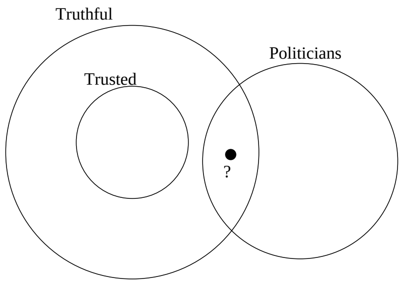
9. (6 points each) Use Euler diagrams to determine whether the following syllogisms are valid or invalid:

(a) All babies are cute.
Some babies cry.
Lenny is not cute
Therefore, Lenny is not a baby.

(b) No politician can be trusted.
Everyone who can be trusted tells the truth.
Therefore, no politicians tell the truth.



Valid



Invalid

10. (5 points each) Define variables and translate each of the following arguments into symbolic form. Then identify the form of each argument and state whether or not the given argument is valid:

(a) If the Colts won the SuperBowl, then I shaved my head. I shaved my head. Therefore, the Colts won the SuperBowl.

Using the variables: p : The Colts win the SuperBowl; q : I shave my head.
The form of this argument is:

$p \to q$
q
―――――
$\therefore p$

The is the Fallacy of the Converse, which is **Invalid**.

(b) If I won a trip to anywhere in the world then I would go to Australia. I did not win a trip to anywhere in the world. Therefore, I did not go to Australia.

Using the variables: p : I won a trip to anywhere in the world; q : I go to Australia.
The form of this argument is:

$p \to q$
$\sim p$
―――――
$\therefore \sim q$

The is the Fallacy of the Inverse, which is **Invalid**.

(c) If I don't quit smoking then I will get lung cancer. I did not get lung cancer. Therefore, I quit smoking.

Using the variables: p : I don't quit smoking; q : I get lung cancer.
The form of this argument is:

$p \to q$
$\sim q$
―――――
$\therefore \sim p$

The is the Law of Contraposition, which is **Valid**.

11. (15 points) Use a truth table to determine whether or not the following argument is valid:

If I get a new job then I will go on vacation.
I will buy a new car or I will go on vacation.
I did not buy a new car.
―――――
Therefore, I got a new job.

Using the variables p : I get a new job; q : I go on vacation; r : I buy a new car, the symbolic form of this argument is:

$p \to q$
$r \vee q$
$\sim r$
―――――
$\therefore p$

Therefore, to analyze the validity of this argument, we will look at the truth table for the statement:
$[(p \to q) \wedge (r \vee q) \wedge (\sim r)] \to p$

p	q	r	$p \to q$	$r \vee q$	$(p \to q) \wedge (r \vee q)$	$\sim r$	$(p \to q) \wedge (r \vee q) \wedge \sim r)$	p	$(p \to q) \wedge (r \vee q) \wedge (\sim r) \to p$
T	T	T	T	T	T	F	F	T	T
T	T	F	T	T	T	T	T	T	T
T	F	T	F	T	F	F	F	T	T
T	F	F	F	F	F	T	F	T	T
F	T	T	T	T	T	F	F	F	T
F	T	F	T	T	T	T	T	F	F
F	F	T	T	T	T	F	F	F	T
F	F	F	T	F	F	T	F	F	T

Notice that there is a False entry in the last column. Therefore, this argument is *invalid.*

12. (10 points) Given the argument:

$p \to r$
$s \to \sim q$
s
$q \vee \sim r$
$\therefore \sim p$

Fill in the missing statements and reasons in the following two column proof:

Statement	Reason
1. $p \to r$	**Premise**
2. $s \to \sim q$	**Premise**
3. s	**Premise**
4. $\sim q$	**2,3, Law of Detachment**
5. $q \vee \sim r$	**Premise**
6. $\sim r$	**5,4, Disjunctive Syllogism**
7. $\sim r \to \sim p$	**1, Contraposition**
8. $\sim p$	**7,6, Law of Detachment**