Math 102
Exam 3 - Version 1 Solutions
11/03/2010

Name:____________________________________

1. (4 points each) Let $U = \{a, b, c, d, e, f, g, h, i, j\}$; $A = \{a, c, d, g, h\}$; $B = \{c, d, e, f, g\}$; $C = \{a, b, c, g, i, j\}$. Give each of the following sets in roster notation:

   (a) $A - C = \{d, h\}$

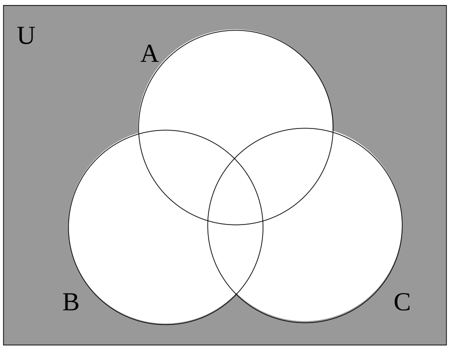
   (b) $A \cup B = \{a, c, d, e, f, g, h\}$

   (c) $(C \cap A') = \{b, i, j\}$

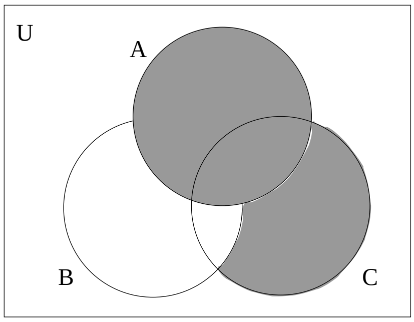
   (d) $(B \cap C)' = \{c, g\}' = \{a, b, d, e, f, h, i, j\}$

2. (5 points each) Illustrate the following by shading the appropriate regions of the given Venn diagrams:

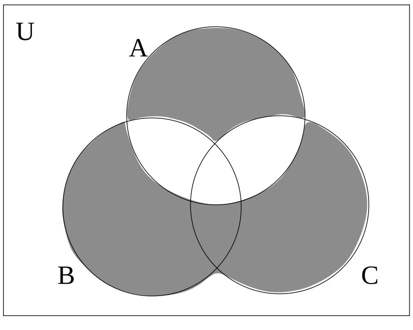
   (a) $(A \cup B \cup C)'$



   (b) $(B' \cap C) \cup (A)$

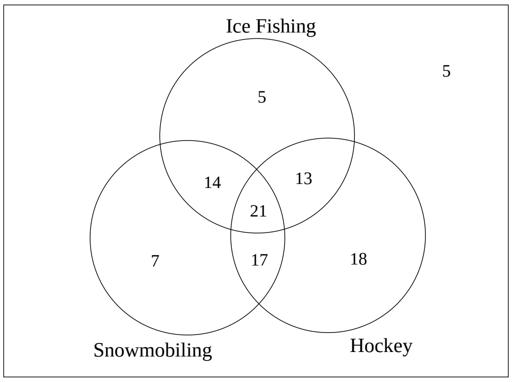


3. (5 points) Use set notation to describe the regions shaded in the Venn diagram given below:



There are several correct ways to describe the set in the Venn diagram above.
Two common ones are: $(A \cup B \cup C) - [(A \cap B) \cup (A \cap C)]$ or $[(B \cup C) - A] \cup [A - (B \cup C)]$

4. (10 points) A survey asked 100 people about winter activities that they enjoy. Specifically, they were asked whether they typically spend time ice fishing, snowmobiling, or playing hockey. Suppose the survey found that 53 of the people surveyed go ice fishing, 69 play hockey, 7 *only* go snowmobiling, 21 do all three, 38 participate in both snowmobiling and hockey, 35 people participate in both snowmobiling and ice fishing, and 35 play hockey but *do not* go ice fishing.



(a) How many do none of the three activities?

5 people do none of the three activities.

(b) How many people go snowmobiling?

14 + 21 + 7 + 17 = 59 people go snowmobiling.

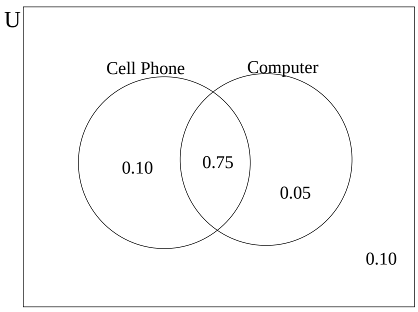
(c) How many go ice fishing but do not play hockey?

53 - 21 - 13 = 19 people go ice fishing but do not play hockey.

(d) How many *only* play hockey?

18 people only play hockey.

5. A survey finds that 85 percent of Americans own a cell phone, 80 percent own a computer, and 75 percent own both a cell phone and a computer.



(a) (4 points) Find the probability that someone owns a cell phone **but not** a computer.

Using the diagram above, $P\left(Cell - Computer\right) = 0.10$.

(b) (4 points) Find the probability that a person owns *neither* a cell phone *nor* a computer.

Using the diagram above, $P\left((Cell \cup Computer)'\right) = 0.10$.

(c) (4 points) *Given* that a person owns a computer, find the probability that this person *also* owns a cell phone.

Using the diagram above, $P\left(Cell|Computer\right) = \frac{.75}{.80} = 0.9375$.

(d) (4 points) Are owning a computer and owning a cell phone independent? Justify your answer.
No. Notice that $P(Cell) = 0.85$ while, from above $P\left(Cell|Computer\right) = 0.9375$. Since the probability of owning a cell phone *changes* when we know that a person has a computer, these two events are **not** independent.

6. A bag contains 7 red balls, 2 green balls, and 3 white balls.

(a) Suppose **one** ball is randomly drawn from the bag (assume each ball is equally likely to be drawn).

i. (5 points) Find the **probability** of drawing a green ball.

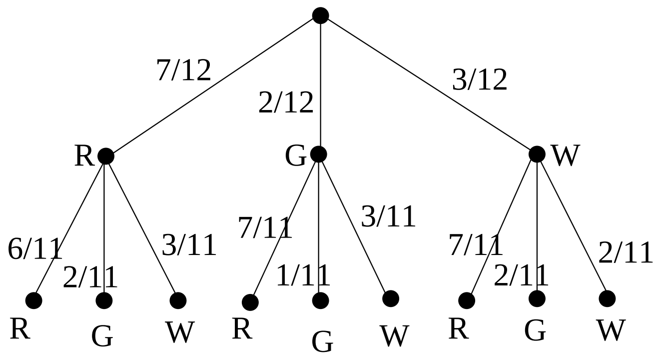
Since there are 12 balls, 2 of which are green, $P(G) = \frac{2}{12} = \frac{1}{6}$.

ii. (5 points) Find the **odds in favor of** drawing a red ball.

Notice that there are 7 red balls and 5 non-red balls. Then the odds in favor of drawing a red ball are $7 : 5$.

(b) Now suppose that all 12 balls are returned to the bag and then **two** balls are randomly drawn from the bag, one at a time, *without replacement.*

The following tree diagram is quite helpful when solving this problem:



i. (5 points) Find the probability that *both* balls are red.

$P(R, R) = \frac{7}{12} \cdot \frac{6}{11} = \frac{42}{132} = \frac{7}{22}$.

ii. (5 points) Find the probability that *neither* ball is white.

There are two main ways to do this problem.
One way is to find the sum of the probabilities for the cases where a non-white ball is chosen at each step:
$P(G, G) + P(G, R) + P(R, G) + P(R, R)$.

A slightly simpler way to compute this is to notice that there are initially 9 non-red balls, so:
$P(R', R') = \frac{9}{12} \cdot \frac{8}{11} = \frac{72}{132} = \frac{6}{11}$.

iii. (5 points) Find the probability that one ball is green and the other is white.

Notice that there are two ways of choosing two balls so that one ball is green and the other is white.
$P(G, W) + P(W, G) = \frac{2}{12} \cdot \frac{3}{11} + \frac{3}{12} \cdot \frac{2}{11} = \frac{6}{132} + \frac{6}{132} = \frac{12}{132} = \frac{1}{11}$.

7. Suppose that we play a game called "Onze". This game is played with 8 identical square tiles with the numbers 1 through 8 printed on one side (the other side is blank). The tiles are thoroughly mixed by the dealer and then are placed face down in two rows of four. The player chooses any two of the tiles and turns them face up.

(a) (5 points) How many different pairs of tiles could the player end up selecting in this game?

This problem is asking us to find the number of objects in the sample space for this game. Notice that we are choosing two of the 8 available tiles, and repetition is not allowed, and that order does not matter. We use combination counting to solve this problem:

$n(S) = C(8,2) = \frac{8!}{6!2!} = \frac{8 \cdot 7}{2} = 28$, so there are 28 different wys of picking two of the eight tiles in this game.

(b) (5 points) Find the *probability* that the total of the numbers on the two tiles is *exactly* 11.

Notice that the only pairs of tiles that add to *exactly* 11 are: 3-8, 4-7, and 5-6. Notice that order does not matter here, so we do not consider 3-8 and 8-3 to be different results.
Let E be the event in which the two tiles picked total *exactly* 11. Then $P(E) = \frac{3}{28}$.

(c) (5 points) Find the *probability* that the total of the numbers on the two tiles is *at least* 11.

Again using counting by listing, the following pairs of tiles total *at least* 11:
3-8, 4-8, 5-8, 6-8, 7-8, 4-7, 5-7, 6-7, 5-6

Let F be the event where the total of the numbers on the two tiles is *at least* 11. Then $P(F) = \frac{9}{28}$

(d) (5 points) Suppose I offer to let you play "Onze" under the following rules: You pay \$1 for the opportunity to play. If you draw two tiles whose numbers total *exactly* 11, then you win \$8 (your original \$1 plus \$7 more). Find the expected value for playing this version of "Onze" (round your answer to the nearest penny). Is it worth your while to play this game?

Recall that there are 28 different pairs of tiles, of which, 3 pairs total *exactly* 11, so the other 25 do not. We assume from the description of the game that if you do not get a total of 11, you lose the dollar you paid to play.
From this, the expected value for this game is:
$E.V. = \frac{3}{28}(\$7) + \frac{25}{28}(-\$1) = \frac{21-25}{28} = \frac{-4}{28} = \frac{-1}{7} \approx -0.14286.$
Therefore, you expect to lose approximately 14 cents per play (on average), so this game *is not* worth playing.

(e) (5 points) Suppose I change the rules of "Onze" as follows: You pay \$1 for the opportunity to play. If you draw two tiles whose numbers total *exactly* 11, then you win \$5 (your original \$1 plus \$4 more). If you draw two tiles whose numbers total *more than* 11, then you win \$3 (your original \$1 plus \$2 more) Find the expected value for playing this version of "Onze" (round your answer to the nearest penny). Is this version better or worse than the previous version (for the player)?

Recall that there are 28 different pairs of tiles, of which, 9 pairs total *at least 11*. Therefore, 6 total *more than* 11 and 3 pairs total *exactly* 11. Thus the other 19 pairs total *less than* 11. We assume from the description of the game that if you get a total of *less than* 11, you lose the dollar you paid to play.
From this, the expected value for this game is:
$E.V. = \frac{3}{28}(\$4) + \frac{6}{28}(\$2) + \frac{19}{28}(-\$1) = \frac{12+12-19}{28} = \frac{5}{28} \approx 0.17857.$
Therefore, you expect to win approximately 17 cents per play (on average), so this game *is* quite a bit better than the previous version (for the player).