

1. Let  $f(x, y) = \sqrt{9 - x^2 - y^2}$ .

- (a) Sketch the domain of  $f$  in the  $x, y$ -plane.  
 (b) Graph contours for  $z = f(x, y)$  for  $z = 0, \sqrt{5}$ , and  $2\sqrt{2}$ .

2. Given the function  $z = f(x, y) = 1 + x^2 - y$ :

- (a) Sketch contours for this function for  $z = 0, 1, 2$   
 (b) What type of curves are the  $x$ -cross sections and the  $y$ -cross sections of  $f$ ?

3. Sketch the domain of the following functions:

(a)  $f(x, y) = \frac{3xy}{y - x^2}$

(b)  $f(x, y) = \sqrt{4 - x^2 - y^2}$

(c)  $f(x, y, z) = \ln(1 - x - y - z)$

4. Compute the following limits:

(a)  $\lim_{(x,y) \rightarrow (2,-1)} \frac{x+y}{x^2 - 2xy}$

(b)  $\lim_{(x,y) \rightarrow (2,-2)} \frac{x+y}{x^2 + xy - x - y}$

(c)  $\lim_{(x,y,z) \rightarrow (1,1,2)} e^{\frac{x+y-z}{x+z}}$

5. Show that the following limits do not exist:

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2 + 2y^2}$

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{y \sin x}{x^2 + y^2}$

(c)  $\lim_{(x,y) \rightarrow (2,0)} \frac{2y^2}{(x-2)^2 + y^2}$

(d)  $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xyz}{x^3 + y^3 + z^3}$

6. Determine all points at which the following functions are continuous:

(a)  $f(x, y) = \ln(3 - x^2 + y)$

(b)  $f(x, y) = \tan(x + y)$

(c)  $f(x, y, z) = 4xe^{y-z}$

7. Let  $f(x, y) = x^2 \sin(xy) - 3y^3$ . Find  $f_x$ ,  $f_y$ ,  $f_{xy}$  and  $f_{yxy}$

8. Let  $f(x, y, z) = x^3y^2 - \sin(yz)$ . Find  $f_{xx}$  and  $f_{yz}$

9. Let  $f(x, y) = 4 - x^2 - y^2$ . Consider the curve  $C$  formed by intersecting  $f$  with the plane  $x = 1$ . Find a parametric equation for the tangent line  $\ell$  to  $C$  at the point  $(1, 1, 2)$ . Then sketch the surface given by  $f$ , the curve  $C$  and the tangent line  $\ell$  on the same graph.

10. Show that the functions  $f_n(x, t) = \sin(n\pi x) \cos(n\pi ct)$  satisfy the wave equation:  $c^2 \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial t^2}$

11. Let  $w = f(x, y) = 2x^2 - xy^2 + 3y$
- Find the increment  $\Delta w$
  - Find the differential  $dw$
  - Find  $dw - \Delta w$
12. Let  $w = f(x, y) = x^2 \ln(y^2)$
- Find  $dw$
  - Use  $dw$  to approximate the change in  $w$  as the input changes from  $(1, 1)$  to  $(1.1, 1.2)$
13. Let  $w = f(x, y) = 4x^2y^3$  where  $x = u^3 - v \sin u$  and  $y = 4u^2 + v$ . Use the Chain Rule to find  $\frac{\partial w}{\partial u}$  and  $\frac{\partial w}{\partial v}$
14. Consider the surface given implicitly by the equation  $xyz - 4y^2z^2 + \cos(xy) = 0$
- Use the Chain Rule to find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$
  - Find an equation for the tangent plane to this surface at the point  $(0, 1, \frac{1}{2})$
15. Recall that when translating from rectangular to polar coordinates  $r = \sqrt{x^2 + y^2}$ .
- Show that  $\frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{r} = \cos \theta$
  - Starting with  $r = \frac{x}{\cos \theta}$ , does it follow that  $\frac{\partial r}{\partial x} = \frac{1}{\cos \theta}$ ? Why or why not?
16. Given that  $w = f(x, y) = x^3 - 2xy$
- Find the equation of the tangent plane to  $f$  at  $(1, -1, 3)$ .
  - Find an equation for the normal line to  $f$  at  $(1, -1, 3)$ .
  - Use the tangent plane you found to estimate  $f(1.1, -0.9)$ . How good is your estimate?
17. Let  $f(x, y) = \sqrt{x^2 + y^2}$
- Find the directional derivative of  $f$  at  $(3, -4)$  in the direction of  $\langle 3, -2 \rangle$ .
  - Find the magnitude and direction of the maximum rate of change of  $f$  at the point  $(3, -4)$ .
18. Find all points at which the tangent plane to the surface  $z = 2x^2 - 4xy + y^4$  is parallel to the  $xy$ -plane.
19. Find  $\nabla F$  at  $(1, 2, 2)$  if  $F(x, y, z) = z^2 e^{2x-y} - 4xz^2$
20. Let  $f(x, y) = x^3 - 3xy + y^3$
- Find all critical points of  $f$ .
  - Classify each critical point using the Discriminant.
21. Let  $f(x, y) = 4xy - x^4 - y^4 + 4$
- Find all critical points of  $f$ .
  - Classify each critical point using the Discriminant.
22. Find the absolute extrema of  $w = f(x, y) = x^2 + y^2 - 2x - 4y$  on the region bounded by  $y = x$ ,  $y = 3$ , and  $x = 0$
23. Find the absolute extrema of  $w = f(x, y) = x^2 + y^2$  on the region bounded by  $(x - 1)^2 + y^2 = 4$