Math 323
LaGrange Multiplier Practice Problems

1. Cascade Container Company produces steel shipping containers at three different plants in amounts x, y, and z, respectively. Their annual revenue is $R(x, y, z) = 2xyz^2$ (in dollars). The company needs to produce 1000 crates annually. How many containers should they produce at each plant in order to maximize their revenue?

2. Use Lagrange multipliers to maximize $f(x, y) = 4x^2y$ subject to the constraint $x^2 + y^2 = 3$.

Solutions:

1. Cascade Cascade Container Company produces steel shipping containers at three different plants in amounts x, y, and z, respectively. Their annual revenue is $R(x, y, z) = 2xyz^2$ (in dollars). The company needs to produce 1000 crates annually. How many containers should they produce at each plant in order to maximize their revenue?

   We will apply the method of La Grange to the function $R(x, y, z) = 2xyz^2$ subject to the constraint $g(x, y, z) = x + y + z = 1,000$.

   Notice that $R_x = 2yz^2$, $R_y = 2xz^2$, $R_z = 4xyz$, and $g_x = g_y = g_z = 1$.

   Therefore, we have $2yz^2 = \lambda = 2xz^2 = 4xyz$. Hence $z = 0$, or $x = y$ and $2yz^2 = 4y^2z$, so $z = 2y$.
   If $z = 0$, then $R(x, y, z) = 0$
   If $x = y$ and $z = 2y$, then $y + y + 2y = 1,000$, or $4y = 1000$, so $x = y = 250$ and $z = 500$.
   In this case, $R(x, y, z) = 2(250)(250)(500)^2 = 31,250,000,000$ dollars of revenue.

2. Use Lagrange multipliers to maximize $f(x, y) = 4x^2y$ subject to the constraint $x^2 + y^2 = 3$.

   We will apply the method of La Grange to the function $f(x, y) = 4x^2y$ subject to the constraint $g(x, y) = x^2+y^2-3 = 0$.

   Notice that $f_x = 8xy$, $f_y = 4x^2$, $g_x = 2x$, and $g_y = 2y$. Also, we must have that $\nabla f = \lambda \nabla g$.
   Therefore, $8xy = 2\lambda x$, so either $x = 0$ or $4y = \lambda$. [Notice that if $x = 0$, $f(x, y) = 0$]
   Using the other pair of partials, $4x^2 = \lambda 2y$, or, substituting, $4x^2 = 8y^2$, or $x^2 = 2y^2$.
   We use this to substitute into the constraint, yielding: $2y^2 + y^2 = 3$, or $3y^3 = 3$, so $y = \pm 1$
   But then $x^2 = 2$, so $x = \pm\sqrt{2}$.
   Finally, $f(\pm\sqrt{2}, 1) = 4(2)(1) = 8$, and $f(\pm\sqrt{2}, -1) = 4(2)(-1) = -8$.
   Hence the maximum of $f(x, y)$ subject to $x^2 + y^2 = 3$ is 8, which occurs at $(\sqrt{2}, 1)$ and $(-\sqrt{2}, 1)$.