Math 323
Quadric Surfaces

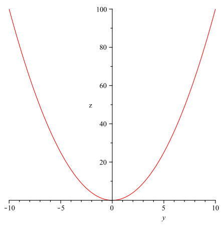
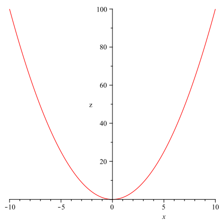
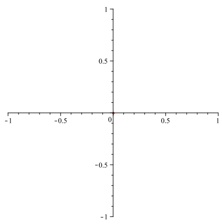
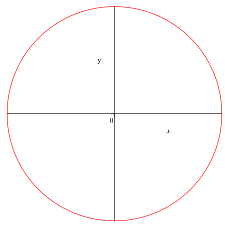
Traces and 3D Graphing:

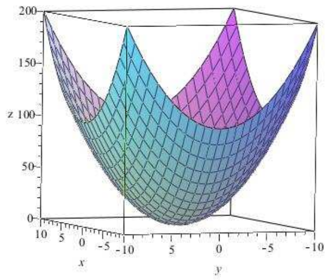
Definition: The *trace* of a graph in 3-dimensional space is the intersection of the graph with a single plane.

• We often think of a trace as a "shadow" or "cross-section" of the graph when it is "sliced" by a particular plane.
• The coordinate planes (the xy, xz, or yz-planes) are often good initial choices to use as planes to look at traces of a 3D graph.
• We can then think of the original graph as being "pieced together" by looking at various 2D traces.

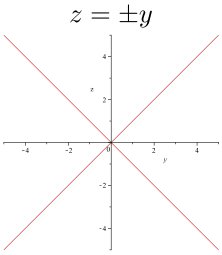
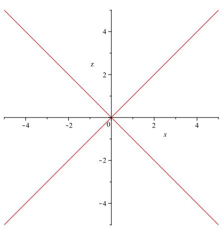
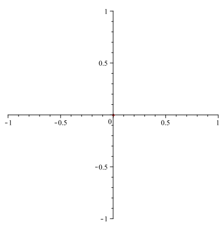
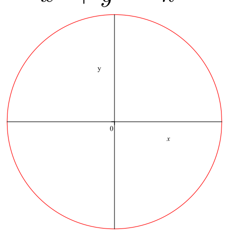
Example 1: Use traces to sketch the graph of $x^2 + y^2 = z$

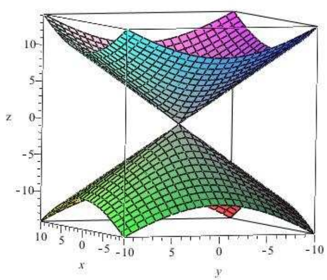
Traces:

$x = 0$	$y = 0$	$z = 0$	$z = k,\ k > 0$
$z = y^2$	$z = x^2$	$x^2 + y^2 = 0$	$x^2 + y^2 = k,\ k > 0$
			



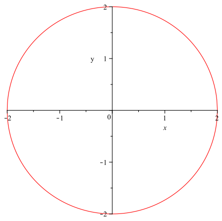
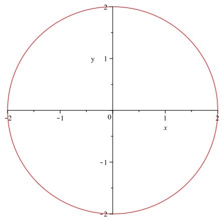
Example 2: Use traces to sketch the graph of $x^2 + y^2 = z^2$

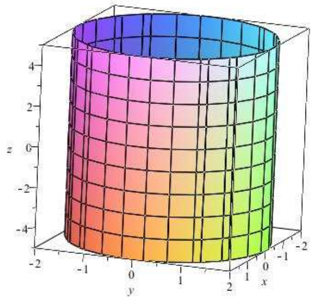
$x = 0$	$y = 0$	$z = 0$	$z = k,\ k > 0$
$z = \pm y$	$z = \pm x$	$x^2 + y^2 = 0$	$x^2 + y^2 = k^2$
			



Example 3: Sketch the graph of $x^2 + y^2 = 4$

Traces:

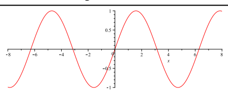
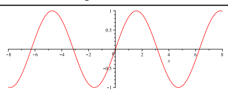
$z = 0$	$z = k$
	

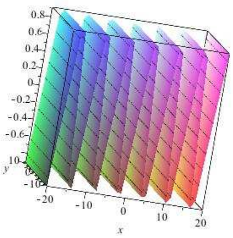


Definition: A *cylinder* is a surface for which there is a plane P that intersects the surface in a curve $\mathcal{C}$, and every place parallel to P has a trace that is equivalent to $\mathcal{C}$. The curve $\mathcal{C}$ is called the *directrix* for the cylinder. Any line perpendicular to P is called a *ruling* of the cylinder.

Example 4: Sketch the graph of $z = \sin x$.

Traces:

$y = 0$	$y = k$
	



Quadric Surfaces:

Definition: A *quadric surface* is a surface that can be represented by an equation of the form:
$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$ for $A, B, C, D, E, F, G, H, I, J$ constants with A, B, C not all zero.

Our goal is to classify all possible quadric surfaces. To simplify things, we will limit our discussion to cases where $D = E = F = G = H = I = 0$. [It turns out that the cases we are leaving out are just translations and rotations of those that we will look at.]