Math 323

Introduction to Vectors

Definitions:

• Any quantity that can be represented by a single real number (along with a related unit of measurement) is called a **scalar quantity**, or, more simply, a **scalar**.

Examples: length, area, volume, speed, and mass are all scalars.

• Any quantitative concept that has both a **magnitude** and a **direction** is called a **vector** and is often represented visually by a directed line segment.

Examples: velocity and force are vectors.

• Two vectors that both have the same magnitude and direction are called **equivalent** vectors.

• We do not usually think of a vector as occupying a particular location, but when a vector is used to describe the motion of a particle from a point P in the plane to another point Q, we draw the vector $\vec{PQ}$ as a directed line segment from P to Q.

• The **position vector** $\vec{OA}$ associated with $\vec{PQ}$ is the vector with the origin as its initial point and with terminal point A chosen so that $\vec{OA}$ and $\vec{PQ}$ are equivalent.

• In order to represent a vector $\vec{a}$ in the plane in a sensible way, we usually compute the horizontal and vertical **components** of the vector $\vec{a}$. These correspond to the x and y coordinates of the point A (the terminal point of the position vector equivalent to $\vec{a}$).

• the **magnitude** or **norm** $\|\vec{a}\|$ of a 2-dimensional vector $\vec{a} = \langle a_1, a_2 \rangle$ is $\|\vec{a}\| = \|\langle a_1, a_2 \rangle\| = \sqrt{(a_1)^2 + (a_2)^2}$

Example: $\|\langle 3, -4 \rangle\| = \sqrt{(3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Addition of Vectors: $\langle a_1, a_2 \rangle + \langle b_1, b_2 \rangle = \langle a_1 + b_1, a_2 + b_2 \rangle$

Scalar Multiplication of Vectors: $c\langle a_1, a_2 \rangle = \langle ca_1, ca_2 \rangle$

Example: $3\langle 3, -4 \rangle - 2\langle -1, 5 \rangle = \langle 9, -12 \rangle + \langle 2, -10 \rangle = \langle 11, -22 \rangle$

The **zero vector** is the vector $\vec{0} = \langle 0, 0 \rangle$

The **negative** of a vector $\vec{a} = \langle a_1, a_2 \rangle$ is the vector $-\vec{a} = \langle -a_1, -a_2 \rangle$. Note that $-\vec{a}$ is a vector with the same magnitude and the opposite direction as $\vec{a}$.

Properties of Vectors:

(i) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$	(ii) $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$	(iii) $\vec{a} + \vec{0} = \vec{a} + \vec{b}$
(iv) $\vec{a} + \vec{-a} = \vec{0}$	(v) $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$	(vi) $(c + d)\vec{a} = c\vec{a} + d\vec{a}$
(vii) $(cd)\vec{a} = c(d\vec{a}) = d(c\vec{a})$	(viii) $1\vec{a} = \vec{a}$	(ix) $0\vec{a} = \vec{0}$

Subtraction of Vectors: $\vec{a} - \vec{b} = \vec{a} + \vec{-b}$

Unit Vectors: A **unit vector** $\vec{u}$ is a vector with **norm** 1. That is, $\|\vec{u}\| = 1$.

Given any vector $\vec{v}$, $\frac{1}{\|\vec{v}\|}\vec{v}$ is a unit vector in the same direction as $\vec{v}$.