

Instructions: You will have until 5:00pm on Monday, December 6th to complete these problems. The points you earn will be added to your score on exam 4.

Given the initial value problem $y' = \frac{\sin(2t) - 2ty}{t^2}$, $1 \leq t \leq 2$, $y(1) = 2$

1. (3 points) Show that this initial value problem is well posed.
2. (2 points) Use Euler's Method to approximate $y(2)$ using $h = 0.5$.
3. (3 points) Use the result of theorem 5.9 to find an upper bound in the error in the approximation for found in part (b). Recall that $|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_{i+1}-a)} - 1]$.
4. (2 points) Use Taylor's Method of order 2 to approximate $y(2)$ using $h = 0.5$.
5. (3 points) Use the Runge-Kutta Order Four method to approximate $y(2)$ using $h = 0.5$.
6. (3 points) Prove that $y(t) = \frac{4 + \cos 2 - \cos(2t)}{2t^2}$ is a solution to this initial value problem.
7. (2 points) Use $y(t) = \frac{4 + \cos 2 - \cos(2t)}{2t^2}$ to compute the actual value of $y(2)$. Then find the absolute error of your previous approximations.

