Math 450
Programming Assignment 2
Due: Friday October 29th

Let $f(x) = \ln x$

1. (a) Find the first five non-zero Taylor Polynomial Approximations of $f(x) = \ln x$ centered at $c = 1$.
   (b) Use the polynomials $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$, and $P_5(x)$ you found above to approximate $\ln 2$. Find the absolute error of each approximation.
   (c) Use maple to produce a plot containing $f(x)$ along with $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$, and $P_5(x)$ [All of these functions should be displayed on the **same** plot]

2. (a) Given $x_0 = 1, x_1 = 1.4, x_2 = 1.8, x_3 = 2.2, x_4 = 2.6, x_5 = 3.0$ use whichever computational method you prefer, find the Lagrange Polynomials $P_{0,1}(x)$, $P_{0,1,2}(x)$, $P_{0,1,2,3}(x)$, $P_{0,1,2,3,4}(x)$, and $P_{0,1,2,3,4,5}(x)$
   (b) Use the polynomials $P_{0,1}(x)$, $P_{0,1,2}(x)$, $P_{0,1,2,3}(x)$, $P_{0,1,2,3,4}(x)$, and $P_{0,1,2,3,4,5}(x)$ you found above to approximate $\ln 2$. Find the absolute error of each approximation.
   (c) Use maple to produce a plot containing $f(x)$ along with $P_{0,1}(x)$, $P_{0,1,2}(x)$, $P_{0,1,2,3}(x)$, $P_{0,1,2,3,4}(x)$, and $P_{0,1,2,3,4,5}(x)$ [All of these functions should be displayed on the **same** plot]

3. (a) Given $x_0 = 1, x_1 = 1.8, x_2 = 2.6$ use the fact that $f'(x) = \dfrac{1}{x}$ to compute $f'(x_0)$, $f'(x_1)$, and $f'(x_2)$. Then, using any appropriate computational method, find a polynomial that agrees with both $f(x)$ and $f'(x)$ at $x_0$, $x_1$, and $x_2$.
   (b) Use the polynomial you found above to approximate $\ln 2$. Find the absolute error of the approximation.
   (c) Use maple to produce a plot containing $f(x)$ and the polynomial you found above.

4. (a) Given $x_0 = 1, x_1 = 1.8, x_2 = 2.6$ find the free spline agreeing with $f(x)$ at these values.
   (b) Given $x_0 = 1, x_1 = 1.8, x_2 = 2.6$ find the clamped spline agreeing with $f(x)$ at these values.
   (c) Use the piecewise defined functions you found above to approximate $\ln 2$. Find the absolute error of each approximation.
   (d) Use maple to produce a plot containing $f(x)$ and the functions you found above.

5. (a) Which of the approximating functions you found above approximate $f(x) = \ln x$ most accurately at $x = 2$?
   (b) Which of the approximating functions you found above approximate $f(x) = \ln x$ most accurately throughout the entire interval $[1, 3]$? Justify your answer.