Math 310
Exam 3 Review Sheet

Section 8.3 Representing Relations

• Understand how to represent a relation using matrices and graphs.
• Given a specific relation, be able to represent it using a matrix and be able to represent is using a directed graph.
• Given a matrix or a directed graph, be able to list the ordered pairs in the relation represented by the matrix (or directed graph).
• Be able to recognize whether a relation on a set is reflexive, irreflexive, symmetric, antisymmetric, or transitive by looking at its matrix.
• Be able to recognize whether a relation on a set is reflexive, irreflexive, symmetric, antisymmetric, or transitive by looking at its directed graph.
• Be able to apply the operations union, intersection, complementation, difference, symmetric difference, and composition to matrices of relations and to directed graphs of relations.

Section 8.5: Equivalence Relations

• Know the definition of an equivalence relation and be able to prove whether a given relation is an equivalence relation.
• Understand the definition of the equivalence class of an element with respect to an equivalence relations on a set A and be able to find the equivalence class of a given element.
• Understand the relationship between equivalence classes and partitions of the underlying set. Be also to tell whether or not a given family of subsets is a partition, and if so, be able find the equivalence relation described by the partition.
• Given a directed graph or a matrix, be able to determine whether the associated binary relation is an equivalence relation.

Section 8.6: Partial Orderings

• Know the definition of a partial order and be able to prove whether a given relation is a partial order.
• Understand the definition of comparable and incomparable elements, and be able to determine whether a given pair of elements are comparable or incomparable with respect to a particular partial order.
• Understand definition of a total ordered set and a well ordered set.
• Understand the lexicographic order, and be able to order a given pair of elements with respect to this ordering.
• Be able to draw the Hasse diagram for a poset. Also be able to read off the ordered pairs in a partial order from its Hasse diagram.
• Understand the definition of maximal elements, minimal elements, greatest elements, least elements, upper bounds, lower bounds, least upper bounds, greatest lower bounds, and the definition of a lattice. Also be able to find these (when they exist) in a given poset.

Section 9.1 - Graphs

• Understand the definition of a graph, and be clear on the difference between simple graphs, multigraphs, pseudographs, directed graphs, etc.
• Given a graph, be able to classify it according to the categories given above.
• Understand how various types of graphs can be used to model real world applications, and be able to choose the most appropriate type of model for a given application.

Section 9.2 - Graph Terminology and Special Types of Graphs

• Understand the definition of adjacency, edge incidence, vertex degree (in-degree and out-degree in directed graphs) and other basic graph terminology.
• Given a graph $\Gamma = (V, E)$, be able to find the cardinality of vertex and edge sets and the degrees of vertices (in-degree and out-degree in directed graphs).
• Memorize the Handshaking Theorem, and be able to use it to prove that an undirected graph has an even number of vertices of odd degree.
• Know the special simple graphs: K_n, C_n, W_n, Q_n, and $K_{m,n}$. Be able to draw examples of these classes of graphs and be able to compute the number of vertices and edges in these classes of graphs.
• Understand the definition of a bipartite graph and be able to prove whether or not a given graph is bipartite.
• Be able to use bipartite graphs to solve matching problems.
• Understand how to form subgraphs of a graph, the union of two graphs, and how to find the complement of a simple graph.

Section 9.3 - Representing Graphs and Graph Isomorphism

• Understand how to represent both directed and undirected graphs using Adjacency Lists, Adjacency Matrices, and Incidence Matrices.
• Understand the definition of graph isomorphism. Also, know the definition of a graph invariant, and key examples of graph invariants ($|E|, |V|$, degree sequences, etc.).
• Given a pair of graphs, be able to prove whether or not the given pair are isomorphic.

Section 9.4 - Connectivity

• Know the definition of a path of length n, a circuit, the sequence of a path, the vertex sequence of a path in a simple graph, and a directed path.
• Understand how paths can be interpreted in various graph models.
• Understand what it means for a graph to be connected, strongly connected, and weakly connected. Also, given a specific graph, be able to determine whether or not it is connected, and if not, be able to find its connected components (or strongly connected components).
• Understand the path length is a graph invariant, and be able to use this fact in graph isomorphism proofs.

Section 9.5 Euler and Hamilton Paths

• Understand the definition of Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Also understand how these are used in graph models.
• Given a specific (di)graph, be able to determine whether or not the graph has an Euler Path or an Euler Circuit by using key theorems. Also be able to find and label them in a graph when they exist.
• Given a specific (di)graph, be able to determine whether or not the graph has an Hamilton Path or a Hamilton Circuit by using key theorems. Also be able to find and label them in a graph when they exist.
• Understand the definition of and be able to identify cut vertices and bridge edges in a given graph.