

Rules of Inference

| Rule of Inference | Tautology | Name |
|--|--|---|
| p $p \rightarrow q$ $\therefore q$ | $[p \wedge (p \rightarrow q)] \rightarrow q$ | Modus Ponens (or Law of Detachment) |
| $\neg q$ $p \rightarrow q$ $\therefore \neg p$ | $[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$ | Modus Tollens (or Law of Contraposition) |
| $p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$ | $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ | Hypothetical Syllogism (or Law of Syllogism) |
| $p \vee q$ $\neg p$ $\therefore q$ | $[(p \vee q) \wedge \neg p] \rightarrow q$ | Disjunctive Syllogism |
| p $\therefore p \vee q$ | $p \rightarrow (p \vee q)$ | Addition |
| $p \wedge q$ $\therefore p$ | $(p \wedge q) \rightarrow p$ | Simplification |
| p q $\therefore p \wedge q$ | $[(p) \wedge (q)] \rightarrow (p \wedge q)$ | Conjunction |
| $p \vee q$ $\neg p \vee r$ $\therefore q \vee r$ | $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$ | Resolution |

Rules of Inference for Quantified Statements

| Rule of Inference | Name |
|--|----------------------------|
| $\forall x P(x)$ $\therefore P(c)$ | Universal Instantiation |
| $P(c)$ for an arbitrary c $\therefore \forall x P(x)$ | Universal Generalization |
| $\exists x P(x)$ $\therefore P(c)$ for some c | Existential Instantiation |
| $P(c)$ for some c $\therefore \exists x P(x)$ | Existential Generalization |