Exam 2 - Practice Problems

- 1. Given the vectors $\vec{a} = \langle 2, 0, -1 \rangle$, $\vec{b} = \langle 3, -2, 4 \rangle$, and $\vec{c} = \langle 1, -4, 0 \rangle$, compute the following:
 - (a) A unit vector in the direction of \vec{c} .
 - (b) The angle between \vec{a} and \vec{b} (to the nearest tenth of a degree).
 - (c) $\vec{a} \times \vec{b}$
 - (d) The area of the triangle determined by \vec{a} and \vec{b} .
 - (e) $com p_{\vec{b}} \vec{a}$.
 - (f) A non-zero vector that is perpendicular to \vec{c} .
- 2. Given the vectors $\vec{a} = \langle 1, 2, 0 \rangle$, $\vec{b} = \langle -1, 0, 2 \rangle$, and $\vec{c} = \langle 0, 1, 1 \rangle$, compute the following:
 - (a) $2\vec{a} + 3\vec{b}$
 - (b) $\vec{a} \cdot \vec{b}$.
 - (c) A unit vector in the direction of \vec{a} .
 - (d) The component of \vec{c} along \vec{a} .
 - (e) The Volume of the parallelepiped determined by \vec{a}, \vec{b} and \vec{c} .
 - (f) The angle between \vec{b} and \vec{c}
- 3. Decide whether each of the following are true or false. If true, explain why. If false, give a counterexample.
 - (a) If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then $\vec{b} = \vec{c}$.
 - (b) If $\vec{b} = \vec{c}$ then $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$.
 - (c) $\vec{a} \cdot \vec{a} = ||\vec{a}||$
 - (d) If $\|\vec{a}\| > \|\vec{b}\|$, then $\vec{a} \cdot \vec{c} > \vec{b} \cdot \vec{c}$
 - (e) If $\|\vec{a}\| = \|\vec{b}\|$, then $\vec{a} = \vec{b}$
- 4. Let P=(2,1,2) and Q=(3,1,1) be points.
 - (a) Find parametric equations for a line through these points.
 - (b) Find the equation of a plane containing P,Q, and the origin.
- 5. Given the lines $l_1: x = 3$; y = 6 2t; z = 3t + 1 $l_2: x = 1 + 2s$; y = 3 + s; z = 2 + 2s:
 - (a) Show that the lines intersect by finding the coordinates of a point of intersection.
 - (b) Find a vector orthogonal to both lines.
 - (c) Find the equation of a plane containing both lines.
- 6. Given the two planes 2x + y z = 4 and 3x y + z = 6
 - (a) Find normal vectors to each plane.
 - (b) Find parametric equations for the line of intersection of the two planes.
- 7. Find both the parametric and symmetric equations for the line through the points P(3,5,7) and Q(-6,2,1).
- 8. Find an equation of the plane through the point (1,4,-5) and parallel to the plane defined by 2x-5y+7z=12.

- 9. Sketch at least 3 traces, then sketch and identify the surface given by each of the following equations:
 - (a) $y^2 + z^2 x = 0$
 - (b) $y^2 + z^2 = 1$
 - (c) $4x^2 + 4y^2 2z^2 = 4$.
 - (d) $4x^2 + y^2 z^2 = 0$
 - (e) $y^2 4 = 4x^2 + z^2$
 - (f) $x^2 z = y^2$
- 10. Let $\vec{r}(t) = \langle t^2, 4+3t, 4-3t \rangle$ be a vector valued function. Calculate the following:
 - (a) $\vec{r'}(t)$
 - (b) $\vec{r''}(t)$
 - (c) $\int_0^1 \vec{r}(t)dt$
 - (d) the arc length of $\vec{r}(t)$ for $0 \le t \le 2$. (Just set up the integral, you do not need to evaluate it).
 - (e) Find the values of t for which $\vec{r}(t)$ and $\vec{r'}(t)$ are perpendicular.
- 11. Let $\mathbf{r}(\mathbf{t}) = \langle 3, 4\cos(t), 4\sin(t) \rangle$.
 - (a) Sketch the curve traced out by the vector-valued function $\mathbf{r}(\mathbf{t})$. Indicate the orientation of the curve. What is geometric shape of this curve?
 - (b) Find an equation in terms of t for s(t), the arc length of the curve traced out by $\mathbf{r}(\mathbf{t})$ as a function of t for $t \geq 0$.
 - (c) Find $\mathbf{r}'(\mathbf{t})$.
 - (d) Find an expression for the angle between the vectors $\mathbf{r}(\mathbf{t})$ and $\mathbf{r}'(\mathbf{t})$.
 - (e) Find the force acting on a 5 kilogram object moving along the path given by $\mathbf{r}(\mathbf{t})$, in units of meters and seconds (assume that no other forces are acting on this mass).
 - (f) Find and draw the position and tangent vectors when $t = \pi$. Find 2 different vectors that are normal to $\mathbf{r}(\pi)$ and $\mathbf{r}'(\pi)$. Draw these vectors (carefully labeled) on the same axes (as the position and tangent vectors).
- 12. Let $\vec{v}(t) = \langle t^2 2t, 4t 3, t^3 \rangle$.
 - (a) Find $\vec{a}(t)$
 - (b) If $\vec{r}(0) = \langle 4, -1, 0 \rangle$, find $\vec{r}(t)$.
 - (c) Find the force acting on an object of mass 50kg with position function $\vec{r}(t)$ (in units of meters per second).
 - (d) Find the speed of the object at time t=2.
- 13. Suppose that a projectile is launched with initial velocity $v_0 = 100$ ft/s from a height of 0 feet and at an angle of $\theta = \frac{\pi}{6}$.
 - (a) Assuming that the only force acting on the object is gravity, find the maximum altitude, horizontal range, and speed at impact of this projectile.
 - (b) Find the landing point of this projectile if it weighs 1 pound, is launched due east, and there is a southerly wind force of 4 pounds.