Math 323
Exam 3 - Practice Problems

1. Let $f(x,y) = \sqrt{9-x^2-y^2}$.
   (a) Sketch the domain of f in the x,y-plane.
   (b) Graph contours for $z = f(x,y)$ for $z = 0, \sqrt{5}$,and $2\sqrt{2}$.

2. Given the function $z = f(x,y) = 1 + x^2 - y$:
   (a) Sketch contours for this function for $z = 0, 1, 2$
   (b) What type of curves are the x-cross sections and the y-cross sections of f?

3. Sketch the domain of the following functions:
   (a) $f(x,y) = \dfrac{3xy}{y-x^2}$
   (b) $f(x,y) = \sqrt{4-x^2-y^2}$
   (c) $f(x,y,z) = \ln(1-x-y-z)$

4. Compute the following limits:
   (a) $\lim_{(x,y)\to(2,-1)} \dfrac{x+y}{x^2-2xy}$
   (b) $\lim_{(x,y)\to(2,-2)} \dfrac{x+y}{x^2+xy-x-y}$
   (c) $\lim_{(x,y,z)\to(1,1,2)} e^{\frac{x+y-z}{x+z}}$

5. Show that the following limits do not exist:
   (a) $\lim_{(x,y)\to(0,0)} \dfrac{2xy}{x^2+2y^2}$
   (b) $\lim_{(x,y)\to(0,0)} \dfrac{y\sin x}{x^2+y^2}$
   (c) $\lim_{(x,y)\to(2,0)} \dfrac{2y^2}{(x-2)^2+y^2}$
   (d) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^3+y^3+z^3}$

6. Determine all points at which the following functions are continuous:
   (a) $f(x,y) = \ln(3-x^2+y)$
   (b) $f(x,y) = \tan(x+y)$
   (c) $f(x,y,z) = 4xe^{y-z}$

7. Let $f(x,y) = x^2\sin(xy) - 3y^3$. Find f_x, f_y, f_{xy} and f_{yxy}

8. Let $f(x,y,x) = x^3y^2 - \sin(yz)$. Find f_{xx} and f_{yz}

9. Let $f(x,y) = 4 - x^2 - y^2$. Consider the curve C formed by intersecting f with the plane $x = 1$. Find a parametric equation for the tangent line ℓ to C at the point $(1,1,2)$. Then sketch the surface given by f, the curve C and the tangent line ℓ on the same graph.

10. Show that the functions $f_n(x,t) = \sin(n\pi x)\cos(n\pi ct)$ satisfy the wave equation: $c^2\dfrac{\partial^2 f}{\partial x^2} = \dfrac{\partial^2 f}{\partial t^2}$

11. Let $w = f(x, y) = 2x^2 - xy^2 + 3y$

   (a) Find the increment Δw
   (b) Find the differential dw
   (c) Find $dw - \Delta w$

12. Let $w = f(x, y) = x^2 \ln(y^2)$

   (a) Find dw
   (b) Use dw to approximate the change in w as the input changes from $(1, 1)$ to $(1.1, 1.2)$

13. Let $w = f(x, y) = 4x^2y^3$ where $x = u^3 - v \sin u$ and $y = 4u^2 + v$. Use the Chain Rule to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$

14. Consider the surface given implicitly by the equation $xyz - 4y^2z^2 + \cos(xy) = 0$

   (a) Use the Chain Rule to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$
   (b) Find an equation for the tangent plane to this surface at the point $(0, 1, \frac{1}{2})$

15. Recall that when translating from rectangular to polar coordinates $r = \sqrt{x^2 + y^2}$.

   (a) Show that $\frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{r} = \cos\theta$
   (b) Starting with $r = \frac{x}{\cos\theta}$, does it follow that $\frac{\partial r}{\partial x} = \frac{1}{\cos\theta}$? Why or why not?

16. Given that $w = f(x, y) = x^3 - 2xy$

   (a) Find the equation of the tangent plane to f at $(1, -1, 3)$.
   (b) Find an equation for the normal line to f at $(1, -1, 3)$.
   (c) Use the tangent plane you found to estimate $f(1.1, -.9)$. How good is your estimate?

17. Let $f(x, y) = \sqrt{x^2 + y^2}$

   (a) Find the directional derivative of f at $(3, -4)$ in the direction of $\langle 3, -2\rangle$.
   (b) Find the magnitude and direction of the maximum rate of change of f at the point $(3, -4)$.

18. Find all points at which the tangent plane to the surface $z = 2x^2 - 4xy + y^4$ is parallel to the xy-plane.

19. Find ∇F at $(1, 2, 2)$ if $F(x, y, z) = z^2e^{2x-y} - 4xz^2$

20. Let $f(x, y) = x^3 - 3xy + y^3$

   (a) Find all critical points of f.
   (b) Classify each critical point using the Discriminant.

21. Let $f(x, y) = 4xy - x^4 - y^4 + 4$

   (a) Find all critical points of f.
   (b) Classify each critical point using the Discriminant.

22. Find the absolute extrema of $w = f(x, y) = x^2 + y^2 - 2x - 4y$ on the region bounded by $y = x$, $y = 3$, and $x = 0$

23. Find the absolute extrema of $w = f(x, y) = x^2 + y^2$ on the region bounded by $(x - 1)^2 + y^2 = 4$

24. Cascade Container Company produces steel shipping containers at three different plants in amounts x, y, and z, respectively. Their annual revenue is $R(x, y, z) = 2xyz^2$ (in dollars). The company needs to produce 1000 crates annually. How many containers should they produce at each plant in order to maximize their revenue?

25. Use Lagrange multipliers to maximize $f(x, y) = 4x^2y$ subject to the constraint $x^2 + y^2 = 3$.