Math 323
Exam 5 **Name:**____

Instructions: This exam is a "Take Home" exam. You will have until 5:00pm on Wednesday, December 7th to complete this exam. You MAY NOT consult with classmates (or anyone else for that matter) on this exam. The credit given on each problem will be proportional to the amount of correct work shown. Answers without supporting work will receive little credit.

1. (8 points) Find equations that represent the flow lines of the vector field $F(x, y) = \langle \cos x, 1 \rangle$. Your answer should be in the form $y = f(x) + C$. Then, find the equation for the specific flow line through the point $(0, 4)$.

2. (10 points) A railing runs along a helical staircase that encircles a silo with a radius of 20 feet. Suppose that silo is 90 feet high and the staircase makes exactly 3 complete revolutions as it climbs from the ground to the top of the silo. Find the mass of the railing if the density of the material used to make the railing has a constant density of 5 lbs per foot.

3. (8 points) Evaluate $\int_{\mathcal{C}} \vec{F} \cdot dr$ if $\vec{F}(x, y, z) = \langle 3x^2y^2z^2, 2x^3yz^2 + z\cos y, 2x^3y^2z + \sin y \rangle$ and $\mathcal{C} = \begin{cases} x = t \\ y = t^2 \\ z = t^3 \end{cases}$ for $0 \leq t \leq 2$

4. (8 points) Use Green's Theorem to evaluate $\oint_{\mathcal{C}} y^3\,dx - x^3\,dy$, where $\mathcal{C}$ is the circle $x^2 + y^2 = 9$.

5. (10 points) Use a line integral to calculate the area of the region in the first quadrant bounded by $y = x^3$ and $y = 4x$. Then verify your answer by computing the area using a double integral in rectangular coordinates. [You must use both methods correctly to receive credit on this question]

6. (10 points) Find the mass of surface S with density function $\delta(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ if S is the portion of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 2$.

7. (10 points) Use the Divergence Theorem to evaluate the flux integral $\iint_S \vec{F} \cdot \vec{n}\, dS$, where $\vec{F}(x, y, z) = \langle x^2 - e^{yz}, e^{xz} - y, x^2 \sin y + z \rangle$ and S is the boundary of the solid Q bounded by $z = \sqrt{x^2 + y^2}$ and the plane $z = 4$.

8. (13 points) Verify Stokes' Theorem for the vector field $\vec{F} = \langle 3z, -x, 2y \rangle$ and the surface S given by the paraboloid $z = 9 - x^2 - y^2$ for $z \geq 0$ with upward normal vectors and $\mathcal{C}$ the positively oriented circle $x^2 + y^2 = 9$ that forms the boundary of S in the xy-plane by computing both $\oint_{\mathcal{C}} \vec{F} \cdot dr$ and $\iint_S curl(\vec{F}) \cdot \vec{n}\, dS$.