Math 323

Final Exam Review Sheet

Note: The Final Exam is on Friday, December 9th from 12:00-2:00pm in our usual classroom, Bridges Room 264.

Chapter 13 Plane Curves, Parameterizations, and the Polar Plane

- Given a parametrically defined plane curve, be able to find an equation in x and y whose graph contains the points on the curve, and be able to graph the curve, indicating its orientation.
- Be able to compute the slope of the tangent line to a point on the graph of a smooth plane curve that is parametrically defined. Also be able to find the equation for the tangent and normal lines at the given point.
- Be able to set up and evaluate a definite integral that gives the arc length of a smooth plane curve defined parametrically.
- Be able to set up and evaluate a definite integral representing the surface area of a surface generated by revolving a parametrically defined, smooth plane curve about a line in the plane.
- Understand the polar plane and be able to plot points and accurately sketch the graphs of various polar equations.
- Be able to translate polar equations into rectangular form and rectangular equations into polar form.
- Be able to set up and evaluate integrals representing the area within a polar region or the area either within or between two polar curves.

Chapter 14 Vectors, Lines, Planes, and Surfaces in 3-Space

- Understand the difference between a vector and a scalar and know how to write a vector in terms of its components.
- Know the algebraic properties of vectors, be able to compute the magnitude of a vector, and be able to find a unit vector in the direction of a given vector.
- Be able to solve application problems involving velocity and force vectors.
- Be able to compute the dot product of two vectors and understand the properties of the dot product, including the connection between the dot product and the norm of a vector.
- Be able to compute the angle between a pair of vectors using the dot product. Also be able to detect when two vectors are perpendicular using the dot product.
- Be able to compute the component of a vector in the direction of another vector. Also be able to compute the work done by a constant force vector in moving an object along a position vector.
- Be able to compute the cross product of two vectors and understand the properties of the cross product, including the fact that the cross product of two vectors is a vector perpendicular to both satisfying the right hand rule.
- Be able to detect when two vectors are parallel using the cross product and be able to compute the areas of triangles and parallelograms.
- Understand parametric equations for a line, be able to find an equation for a line from a description of the line. Also be able to find the point of intersection of a pair of lines that intersect.
- Understand what a normal vector to a plane is, and be able to find the equation of a plane when given either a normal vector and a point on the plane, or, given 3 non-colinear points in a plane.
- Be able to determine whether or not a pair of planes are parallel. If not, be able to find an equation for the line of intersection of two planes.
- Understand how to graph cylinders and quadric surfaces by looking at traces.

Chapter 15 Vector Valued Functions and the Calculus of Vector Valued Functions

- Understand vector valued functions and be able to sketch the vectors and curves associated with a vector valued function.
- Be able to find the arc length of the curve determined by a vector valued function on a given interval.
- Understand how to find limits, determine continuity, compute derivatives, and definite integrals of vector valued functions and understand the interpretation of the derivative of a vector valued function as the tangent vector to the associated curve.
- Be able to compute velocity, acceleration, speed, and work in a given situation involving a vector valued function and be able to solve projectile motion application problems in both 2 and 3D.

Chapter 16 Functions of Several Variables

- Understand the definition of a function of several variables, be able to find the domain and range of a function or several variables, and be able to sketch the domain of a function of two or three variables.
- Be able to represent the graph of a function of two variables by understanding, graphing, and labeling level curves of the function as part of a contour plot. Also be able to represent the graph of a function of three variables by understanding, graphing, and labeling level surfaces of the function.
- Understand the definition of the the limit of a function of several variables at a point in its domain.
- Understand the definition of continuity of a function of several variables at a point in its domain and be able to compute the limit of a function of several variables at a point where it is continuous. Also be able to determine the set of points at which a given function of several variables in continuous.
- Be able to show that the limit of a function of several variables does **not** exist by showing that the limit attains different values when computed along two different paths.
- Be able to compute the partial derivatives of a function of several variables as well as higher order partials of the function.

• Understand the partial derivative of a function as the slope of the tangent line to the trace of the function in a given plane.
• Understand that the mixed partials of a continuous function of several variables are equal to each other.
• Know the definition of a differentiable function of several variables and the connection between continuity and differentiability as stated in Theorems 16.17 and 16.18
• Understand the chain rule for functions of one or more variables and be able to apply the chain rule in order to find derivatives or partial derivatives involving the composition of functions of one of more variables. Also be able to use the chain rule to find the derivative or partial derivative of functions that are given implicitly.
• Understand the definition of the gradient of a function and be able to use it to compute directional derivatives. Also understand the geometric meaning of the gradient of a function as the direction of the maximum rate of increase and be able to use the gradient to find the magnitude and direction of both the maximum and minimum rates of changes of the function at a given point.
• Be able to use directional derivatives in order to find tangent lines to a curve and to solve other application problems.
• Understand how to use the gradient of a function to find the equation of the tangent plane to a function of several variables at a given point for functions given both implicitly and explicitly.
• Be able to use the tangent plane to a function in order to approximate the value of a function at a "nearby" point.
• Be able to find the parametric equation of the normal line to a point on the graph of a function of several variables.
• Understand that the gradient vector at a point in the graph of a function of several variables is orthogonal to the level curve (or surface) of the function through that point.
• Understand critical points and local extrema of functions of several variables.
• Be able to find and classify the critical points of a function of several variables using the Discriminant.
• Understand the definition of a saddle point of a function of several variables.
• Understand that a continuous function of several variables has absolute extrema on any closed region in its domain. Be able to find the absolute extrema of a given function by testing its critical points and boundary points.
• Be able to use La Grange's Theorem to find the extrema of a function of several variables subject to a single constraint.

Chapter 17 Integrating Functions of Several Variables

• Understand the definition of a double and triple integrals over a region in the plane and be able to use partitions and a sum of rectangular solids arising form a given partition in order to approximate the value of a double integral.
• Memorize Fubini's Theorem, and be able to apply it to evaluate integrals by rewriting them as iterated integrals.
• Given a description of a region in the plane R , be able to express R using the limits of integration of an iterated integral. Also, given a iterated double integral, be able to graph the region of integration.
• Be able to express and compute the area of a region in the plane or the volume under a given function and over a region in the plane using iterated integrals.
• Understand how translate a double integral over a region in the plane into an iterated polar integral. In particular remember the differential involved: $dA = r\,dr\,d\theta$.
• Be able to set up an iterated integrals representing the areas and volumes in rectangular, polar, cylindrical, and spherical coordinates. Be able to determine the best coordinate system to use for a given integral.
• Be able to translate iterated integrals from one coordinate system into another.
• Understand how represent the surface area of a surface over a region in the plane using a double integral.
• Be able to use a triple integral to find the mass of a solid Q given a density function $\delta(x, y, z)$ for Q.
• Memorize the evaluation theorem for iterated integrals in cylindrical and spherical coordinates, especially the differential and coordinate conversion formulas.
• Given an integral in cylindrical or spherical coordinates, be able to graph the solid Q determined by the limits of integration.

Chapter 18 Vector Calculus

• Understand the definition of a vector field in 2 or 3 dimensions and be able to sketch graphs of 2-dimensional vector fields.
• Know the definition of a conservative vector field and be able to find a potential function for a conservative vector field.
• Be able to compute the curl and divergence of a given vector field.
• Understand the definition of a line integral and be able to use line integrals to compute the mass of a wire and the work done in traversing a path through a given vector field.
• Understand how to determine whether or not a vector field is independent of path and be able to apply the Fundamental Theorem of Line Integrals to compute the work done in moving from one point to another in a conservative vector field.
• Understand what a simple closed curve is, and how to give a closed curve a positive orientation.
• Be able to use Green's Theorem both to translate a work integral along a simple closed curve into a related double integral. Also be able to compute the area of a region using a work integral.
• Be able to evaluate surface integrals for a surface given explicitly by a function of two variables.
• Understand how to express the "outward" or "upward" unit normal vectors along a given surface.
• Be able to set up and compute integrals representing the flux along a surface given by an explicit function of two variables.
• Understand the statement of the Divergence Theorem and be able to use it to translate from a flux integral along a surface enclosing a solid region to a triple integral over the enclosed region and vice versa.
• Understand the statement of Stokes' Theorem and be able to use it to translate from a flux integral along a surface bounded by a simple closed curve to a work integral around the curve and vice versa.