Math 310
Logic and Truth Tables

Definition: A **proposition** is a declarative sentence that is either true or false (but not both). That is, a statement ends up having one of two possible truth values:

- If a statement is true, we assign it the truth value T. If a statement is false, we assign it the truth value F. In practice, we will often think of statements as *potentially* having one of the two possible truth values without actually giving it a specific assignment.
- In fact, we may not even give an actual statement. Instead, we use lower case letters such as p, q, r, s to denote **propositional variables** (unspecified statements).
- We then consider all possible ways of assigning truth values to variables representing statements, often after combining several statements using *logical connectives* to form **compound propositions**.

Basic Truth Tables:

The truth table for "not" ($\neg$): Given a simple statement p. If p is true, then $\neg p$ is false. Similarly, if p is false, then $\neg p$ is true. Note that the statement p has only two possible truth values, so there are two rows in the truth table. The following table summarizes this information.

p	$\neg p$
T	F
F	T

The truth table for "or" ($\vee$): Given two simple statements p, q, the compound statement $p \vee q$ has four possible truth value assignments: both could be true, both could be false, the first could be true while the second is false, and the first could be false while the second is true. Since $\vee$ always represents an *inclusive or*, the statement $p \vee q$ is true except when p and q are both false. The following table summarizes this information.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The truth table for "and" ($\wedge$): Given two simple statements p, q, as above the compound statement $p \wedge q$ has four truth value assignments. The statement $p \wedge q$ is only true when p and q are both true. The following table summarizes this information.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The truth table for a conditional ($\rightarrow$): Given two simple statements p, q, as above the compound statement $p \rightarrow q$ has four possible truth value assignments. The statement $p \rightarrow q$ is true except in the case when p is true and q is false. To see this, it is helpful to think about the conditional statement, "If you eat your vegetables then you will get dessert." When is this a false statement? The following table summarizes this information.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The truth table for a biconditional ($\longleftrightarrow$): Given two simple statements p, q, as above the compound statement $p \longleftrightarrow q$ has four possible truth value assignments. The statement $p \longleftrightarrow q$ is true when p and q are both true and when p and q are both false. When p and q have opposite truth values, the statement $p \longleftrightarrow q$ is false. The following table summarizes this information.

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T