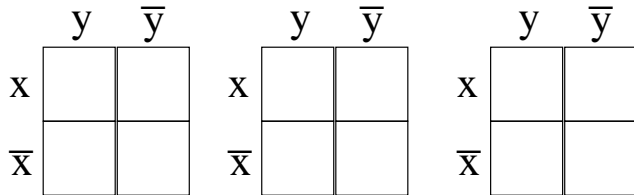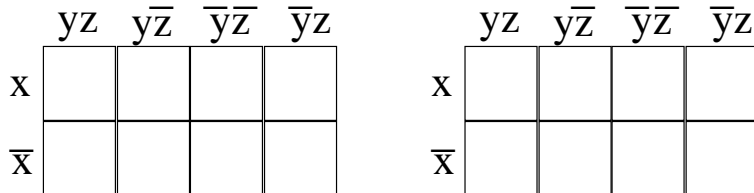Math 310
11.4 Minimizing Circuits

Karnaugh Maps:
A **Karnaugh Map** is a diagrammatical method for finding the minimization for a Boolean expression in disjunctive normal from. To use a K-map to simplify an expression, we put a 1 in the square in the diagram corresponding to each minterm represented in the Boolean expression. We circle all *blocks* in the diagram. Blocks are collections of 2^k adjacent cells for some k. A minimization for the given Boolean expression is found by choosing a collection of *maximal* blocks which *cover* the 1s in the diagram. A minimal expression is found by writing out the terms associated with the collection of blocks chosen.
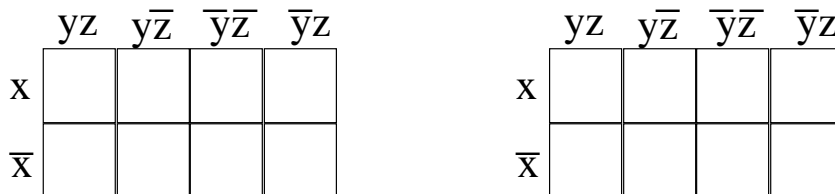
Examples:
    1. $x\overline{y} + \overline{x}\,\overline{y}$ 2. $xy + \overline{x}\,\overline{y}$ 3. $xy + \overline{x}y + \overline{x}\,\overline{y}$

	y	$\overline{y}$
x		
$\overline{x}$		

	y	$\overline{y}$
x		
$\overline{x}$		

	y	$\overline{y}$
x		
$\overline{x}$		

    4. $xy\overline{z} + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}\,\overline{z}$ 5. $x\overline{y}z + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}z + \overline{x}\,\overline{y}\,\overline{z}$

	yz	y$\overline{z}$	$\overline{y}\overline{z}$	$\overline{y}$z
x				
$\overline{x}$				

	yz	y$\overline{z}$	$\overline{y}\overline{z}$	$\overline{y}$z
x				
$\overline{x}$				

    6. $xyz + xy\overline{z} + x\overline{y}z + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}z + \overline{x}\,\overline{y}\,\overline{z}$ 7. $xy\overline{z} + x\overline{y}\,\overline{z} + \overline{x}\,\overline{y}z + \overline{x}\,\overline{y}\,\overline{z}$

	yz	y$\overline{z}$	$\overline{y}\overline{z}$	$\overline{y}$z
x				
$\overline{x}$				

	yz	y$\overline{z}$	$\overline{y}\overline{z}$	$\overline{y}$z
x				
$\overline{x}$				

8. $wx\overline{y}\overline{z} + w\overline{x}yz + w\overline{x}y\overline{z} + w\overline{x}\,\overline{y}\,\overline{z} + \overline{w}x\,\overline{y}\,\overline{z} + \overline{w}\,\overline{x}y\,\overline{z} + \overline{w}\,\overline{x}\,\overline{y}\,\overline{z}$

	yz	y$\overline{z}$	$\overline{y}\overline{z}$	$\overline{y}$z
wx				
w$\overline{x}$				
$\overline{w}\overline{x}$				
$\overline{w}$x				
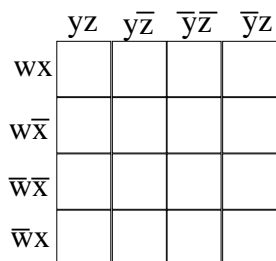
Method 2: The Quine-McClusky Method

K-Maps become very difficult to draw and interpret once we have more than 4-5 variables in our expression. The Quine-McClusky method is an algorithmic procedure that automates the process of finding a minimization of a Boolean Expression. The algorithm proceeds as follows:

1. List each minterm in the given Boolean expression one at a time and convert each minterm to a binary string.

2. Find all pairs of minterms that can be combined into a shorter term (the binary strings for these terms will differ in exactly one position). List these pairs and their strings in a new column.

3. Apply the same process in your new column of shortened strings.

4. Continue this process until all possible terms have been combined. This process results in a list of terms of various length.

5. Work backwards from the shortest terms you found up to the largest terms choosing a minimal collection of terms to generate all of the original minterms (it may be useful to create a separate a table for this step.

Examples: