## Rules of Inference

Rule of Inference	Tautology	Name
p	$[p \land (p \to q)] \to q$	Modus Ponens
$p \rightarrow q$		(or Law of Detachment)
$  \therefore q$		
$\neg q$	$[\neg q \land (p \to q)] \to \neg p$	Modus Tollens
$p \rightarrow q$		(or Law of Contraposition)
$  \therefore \neg p$		
$p \rightarrow q$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical Syllogism
$q \rightarrow r$		(or Law of Syllogism)
$\therefore p \to r$		
$p \lor q$	$[(p \lor q) \land \neg p] \to q$	Disjunctive Syllogism
$ \neg p $		
$\therefore q$		
$\mid p$	$p \to (p \lor q)$	Addition
$\therefore p \lor q$		
$p \wedge q$	$(p \land q) \to p$	Simplification
$\therefore p$		
p	$[(p) \land (q)] \to (p \land q)$	Conjunction
$\mid q$		
$\therefore p \land q$		
$p \lor q$	$[(p \lor q) \land (\neg p \lor r)] \to (q \lor r)$	Resolution
$\neg p \lor r$		
$\therefore q \vee r$		

## Rules of Inference for Quantified Statements

Rule of Inference	Name	
$\forall x P(x)$	Universal Instantiation	
$\therefore P(c)$		
P(c) for an arbitrary $c$	Universal Generalization	
$\therefore \forall x P(x)$		
$\exists x P(x)$	Existential Instantiation	
$\therefore P(c)$ for some $c$		
P(c) for some $c$	Existential Generalization	
$\therefore \exists x P(x)$		