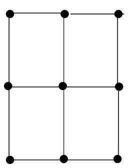
Math 310
Project 2 Handout
Due: Friday, December 7th by 4:00pm

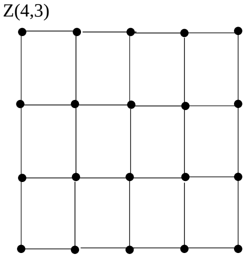
Instructions: This project is designed to give you an opportunity to explore some of the concepts from class in a little more depth. You may work with **at most two** other students on this assignment. If you decide to work with other students, you may turn in a combined paper with each of your names listed.

Let $Z(m, n)$ be the graph enclosing a grid of $m \times n$ squares in the plane. For example:

Z(2,2)



Z(4,3)



1. If $\Gamma = Z(m, n)$:
   (a) (2 points) Find an expression for $|V|$
   (Your answer to this question and many questions that follow will be in terms of m and n).
   (b) (2 points) Find an expression for $|E|$.
   (c) (2 points) Since $\Gamma = Z(m, n)$ is planar for all m and n, find an expression for the number of regions in a planar drawing of $Z(m, n)$.
   (d) (2 points) Find an expression for the number of vertices v with $deg(v) = 2$.
   (e) (2 points) Find an expression for the number of vertices v with $deg(v) = 3$.
   (f) (2 points) Find an expression for the number of vertices v with $deg(v) = 4$.
2. (2 points) For which m and n does $Z(m, n)$ have an Euler Circuit? Justify your answer.
3. (2 points) For which m and n does $Z(m, n)$ have an Euler Path? Justify your answer.
4. (2 points) For which m and n does $Z(m, n)$ have a Hamilton Circuit? Justify your answer.
5. (2 points) For which m and n does $Z(m, n)$ have a Hamilton Path? Justify your answer.
6. (2 points) Find the vertex chromatic number for each $Z(m, n)$ graph.
7. (3 points) Find the edge chromatic number for each $Z(m, n)$ graph.

Extra Credit:
If a graph does not have an Euler Circuit, one can still try to find the most efficient way of traversing all the edges of the graph with the minimum possible number of "retracings". To do this, we consider all of the "Eulerizations" of the graph. To "Eulerize" a graph, one adds multiedges to the graph (copies of **existing edges** representing retracing that edge) until we get a new graph whose vertices all have even degree. An **optimal Eulerization** for a graph is an Eulerization that has the fewest possible extra edges. Find the number of edges needed to produce an **Optimal Eulerization** for as many $Z(m, n)$ graphs as you can.
I will give 1 point for finding an optimal Eulerization for a reasonably complicated example, but to earn further points, you will need to show how to compute the number of edges needed to produce an optimal Eulerization for many $Z(m, n)$ graphs using some sort of general method. Try to find ways to handle large classes of $Z(m, n)$ graphs.