Math 450
Programming Assignment 2
Due: Monday October 29th

1. Let $f(x) = \ln x$.

   (a) Find (either directly or in Maple) the first five non-zero Taylor Polynomial Approximations of $f(x) = \ln x$ centered at $c = 1$.

   (b) Use the polynomials $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$, and $P_5(x)$ you found above to approximate $\ln 2$. Find the absolute error of each approximation.

   (c) Use Maple to produce a plot containing $f(x)$ along with $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$, and $P_5(x)$ [All of these functions should be displayed on the **same** plot]

2. (a) Write a program that, when given as input a set of data values $(x_0, y_0), (x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$, outputs the Lagrange Polynomial of degree n passing through these points. [To make things easier, since we know the underlying function being used, you may set your program up so that the input is in the form: $(n, f(x), x_0, x_1, \cdots, x_n)$. Since the data below is equally spaced, you could choose to write a program accepting equally spaced data taking input: $(n, f(x), x_0, h = \Delta x)$].

   (b) Let $f(x) = \ln x$ and let $x_0 = 1, x_1 = 1.4, x_2 = 1.8, x_3 = 2.2, x_4 = 2.6, x_5 = 3.0$. Use your program to find the Lagrange Polynomials $P_{0,1}(x)$, $P_{0,1,2}(x)$, $P_{0,1,2,3}(x)$, $P_{0,1,2,3,4}(x)$, and $P_{0,1,2,3,4,5}(x)$.

   (c) Use the polynomials $P_{0,1}(x)$, $P_{0,1,2}(x)$, $P_{0,1,2,3}(x)$, $P_{0,1,2,3,4}(x)$, and $P_{0,1,2,3,4,5}(x)$ you found above to approximate $\ln 2$. Find the absolute error of each approximation.

   (d) Use Maple to produce a plot containing $f(x)$ along with $P_{0,1}(x)$, $P_{0,1,2}(x)$, $P_{0,1,2,3}(x)$, $P_{0,1,2,3,4}(x)$, and $P_{0,1,2,3,4,5}(x)$ [All of these functions should be displayed on the **same** plot]

3. (a) Write a program that, when given input $(n, f(x), f'(x), x_0, x_1, \cdots, x_n)$, computes the associated Hermite Polynomial. [You may use either method that we know for computing Hermite Polynomials as the basis for your program. Notice that I am allowing you to give $f'(x)$ as part of the input].

   (b) Let $f(x) = \ln x$ and let $x_0 = 1, x_1 = 1.8, x_2 = 2.6$. Use your program to find the Hermite polynomial that agrees with both $f(x)$ and $f'(x)$ at x_0, x_1, and x_2.

   (c) Use the polynomial you found above to approximate $\ln 2$. Find the absolute error of the approximation.

   (d) Use Maple to produce a plot containing $f(x)$ and the polynomial you found above.

4. (a) Given $x_0 = 1, x_1 = 1.8, x_2 = 2.6$ find the free spline agreeing with $f(x)$ at these values.

   (b) Given $x_0 = 1, x_1 = 1.8, x_2 = 2.6$ find the clamped spline agreeing with $f(x)$ at these values.

   [**Note:** unlike the previous parts, you may find these via direct computation in Maple rather than using a program written in Maple or another programming language.]

   (c) Use the piecewise defined functions you found above to approximate $\ln 2$. Find the absolute error of each approximation.

   (d) Use Maple to produce a plot containing $f(x)$ and the functions you found above.

   (e) **Extra credit:** Write a program to find the free spline fitting data input as: $(n, f(x), x_0, x_1, \cdots, x_n)$.

5. (a) Which of the approximating functions you found above approximate $f(x) = \ln x$ most accurately at $x = 2$? Justify your answer.

   (b) Which of the approximating functions you found above approximate $f(x) = \ln x$ most accurately throughout the entire interval $[1, 3]$? Justify your answer.