Math 476
Exam 1 Review Sheet

Chapter 0: Preliminaries

• Be able to do proof by induction.
• Know and be able to apply the Well Ordering Principle for sets of positive integers.
• Know the statement of Theorem 0.1 (The Division Algorithm) and be able to apply it.
• Know the statement of Theorem 0.2 and its Corollary.
• Know the statements of Theorem 0.3 (The Fundamental Theorem of Arithmetic) and Euclid's Lemma and be able to apply them.
• Know the definitions of the terms: greatest common divisor, least common multiple, prime, and relatively prime.
• Understand Modular Arithmetic and be able to carry out addition and multiplication mod n.

Chapter 1: Introduction to Groups

• Understand the group D_4 and be able to build and apply the Cayley Table for this group.
• Know the definition of general dihedral groups D_n (the symmetry group of a regular n-gon).

Chapter 2: Binary Operations and Groups

• Know the definition of a binary operation.
• Understand the definition of associativity, commutativity, identity element, the inverse of an element, and a zero element and be able to determine whether or not which of these are satisfied or present in a given binary operation on a set.
• Know the definition of a group and be able to determine whether or not a binary operation on a set is a group.
• Be able to construct and/or use the Cayley Table for a binary operation.
• Be familiar with all of the examples of groups given in Table 2.1 in your textbook.
• Know the statements and proofs of Theorems 2.1, 2.2, 2.3, and 2.4 and be able to use them to carry out basic computations in a group.

Chapter 3: Finite Groups and Subgroups

• Know the definition of the order of a group and the order of an element of a group. Also be able to carry out proofs and computations related to these definitions.
• Know the definition of a subgroup of a group.
• Know the statements of Theorems 3.1, 3.2, and 3.3 and be able to apply there theorems in order to determine whether or not a subset of a group is a subgroup.
• Know the statements and proofs of Theorems 3.4 and 3.5.
• Know the definitions of $Z(G)$ and $C(a)$ and be able to find the elements in these sets for a given group and a given element.

Chapter 4: Cyclic Groups

• Know the statement of Theorem 4.1 and its Corollaries.
• Know the statement of Theorem 4.2 and its Corollaries.
• Know the statement of Theorem 4.3 and its Corollary.
• Be able to apply these results to find all generators for a given cyclic group.
• Be able to apply there results to find all possible orders of elements in a given cyclic group.
• Be able to find all subgroups of a cyclic group and construct the subgroup lattice.
• Understand the definition of Euler's phi function and be able to use it and Theorem 4.4 to find the number of elements of a given order in a cyclic group.

Chapter 5: Permutation Groups

• Understand the definition of a permutation and the definition of a permutation group.
• Be able to represent a permutation in S_n in both double row and disjoint cycle notation.
• Be able to find the product of two (or more) permutations.
• Be able to write a permutation as a product of 2-cycles.
• Be able to find the order of a given permutation.
• Know the proof of Theorem 5.2 and the statements of Theorems 5.1, 5.3, 5.4, 5.5, and 5.7.
• Know the definition of even and odd permutations and be able to determine whether a given permutation is even or odd.
• Be able to generate a list of all possible orders of elements in S_n for a given n.
• Be able to determine the number of elements of a given order in S_n for a given n.
• Know the definition of A_n and be able to show that A_n is a subgroup for any n.