

Chapter 6: Isomorphisms

- Know the definition of group isomorphism. Also know and be able to apply the 4-step process for verifying that a function is an isomorphism.
- Be able to show that two groups are isomorphic by finding an isomorphism between them.
- Be able to show that a given function is not an isomorphism. Also be able to show that two groups are not isomorphic by showing that it is not possible to construct an isomorphism between them.
- Know the statement of Cayley's Theorem (Theorem 6.1). Also know the definition of the left regular representation of a group used in the proof of Cayley's Theorem.
- Know the statements of Theorems 6.2 and 6.3 well enough to be able to apply them (you don't need to know all of the parts word for word, but several of these properties will be used in one way or another on the exam). It is also highly likely that I will ask you to prove one or two of the properties (I will give you the statement of the property in question).
- Know the definition of an automorphism and an inner automorphism. Also know the statement and proof of Theorem 6.4.

Chapter 7: Cosets and Lagrange's Theorem

- Know the definition of the left (right) cosets of a subgroup H of a group G . Also be able to find the cosets in a particular example.
- Know the properties of cosets in the main Lemma from page 139 in the textbook. It is likely that I will ask you to prove one or two of these properties.
- Know the statement of Lagrange's Theorem and its first 4 corollaries and be able to apply them.
- Know that the converse of Lagrange's theorem is false (also know a specific counterexample).
- Know the definitions of the stabilizer of a point and the orbit of a point. Know and be able to apply Theorem 7.3.

Chapter 8: External Direct Products

- Know the definition of the external direct product of a finite number of groups. Also know and be able to apply Theorem 8.1 to compute the order of an element in the direct product of two groups and to help you count the number of elements of a given order in the direct product of two groups.
- Know and be able to apply Theorem 8.2 and its two corollaries.

Chapter 9: Normal Subgroups and Factor Groups

- Know the definition of a normal subgroup of a group G . Also know Theorem 9.1 and be able to apply it or the definition to determine whether or not a subgroup of a given group is normal.
- Know examples 1, 2 and 3 from page 179 in your textbook.
- Know the definition of a factor group. Know and be able to prove Theorem 9.2.
- Be able to build a Cayley table for a specific factor group and be able to carry out computations in a given factor group.
- Know the statement and proof of Theorem 9.3. Also know the statements of Theorems 9.4, 9.5, and 9.7.

Chapter 10: Group Homomorphisms

- Know the definition of the order of a group homomorphism. Also be able to determine whether or not a given function between two groups is a homomorphism.
- Know the definition of the kernel of a homomorphism. Also be able to find the kernel of a given homomorphism.
- Know the statements of Theorems 10.1 and 10.2, as well as the corollary to Theorem 10.2. It is likely that I will ask you to prove one or two of these properties.
- Be able to determine all homomorphisms between two groups \mathbb{Z}_n and \mathbb{Z}_m and be able to investigate the properties of a specific homomorphism of this form.
- Know the statement and proof of Theorem 10.3 and the statement of Theorem 10.4.

Chapter 11: The Fundamental Theorem of Finite Abelian Groups

- Know the statement of Theorem 11.1 and be able to apply it to determine the number of isomorphism classes of abelian groups of a given order.
- Also be able to determine which isomorphism class a particular example of a given group falls into by analyzing the orders of the elements of the group and applying Theorem 11.1.
- Know the statement of the Corollary to Theorem 11.1 and be able to apply it.

Note: You are also expected to remember and be able to apply key concepts and definitions from the previous exam. Although they will not be tested explicitly, properties of groups, subgroups, cyclic groups, and permutation groups will come up incidentally in the material on this exam.