Math 476
Exam 4 Review Sheet

Chapter 15: Ring Homomorphisms and Fields of Quotients

• Know the definition of a ring homomorphism and be able to determine whether or not a function between two rings is a homomorphism.
• Know the statement of Theorem 15.5 and the statements of its Corollaries. Be able to apply these results to specific rings.
• Know the the statement and proof of Theorem 15.6. I will not make you for the entire proof, but I may have prove one or more of the properties needed to verify that the construction of a field of quotients is well defined or that the resulting object is indeed a field.
• Be able to carry out computations in a field of quotients.

Chapter 16: Polynomial Rings

• Memorize the definition of a ring of polynomials $R[x]$ over a commutative ring R, including the definition of the operations in $R[x]$.
• Know the definition of: the degree of a polynomial, leading term, leading coefficient, constant term, and monic polynomial.
• Know the statements of Theorem 16.1 and 16.2(The Division Algorithm). Also be able to carry out the division algorithm.
• Know the statements of the three Corollaries to Theorem 16.2.
• Know the definition of a principal ideal domain. Also know the statement and proof of Theorem 16.3 and the statement of Theorem 16.4

Chapter 17: Factorization of Polynomials

• Know the definition of reducible and irreducible polynomials in an integral domain D.
• Know both the statement and proof of Theorem 17.1
• Know the definition of the content of a polynomial and the statement of Gauss's Lemma.
• Know the statement of Theorem 17.2, the statement and proof of Theorem 17.3 (Mod p irreducibility) , and the statement of Theorem 17.4 (Eisenstein's Criterion).
• Given a polynomial in $\mathbb{Z}[x]$ or $\mathbb{Q}[x]$, be able to use irreducibility tests to show that the polynomial is irreducible.
• Given a polynomial in $\mathbb{Z}_p[x]$ be able to either use irreducibility tests to show that the polynomial is irreducible, or be able to fully factor the polynomial.
• Know the statement of Theorem 17.5 and the statement of both of its corollaries. Be able to use Corollary 1 to construct a finite field.

Chapter 20: Extension Fields

• Know the definition of an extension field of a field $\mathbb{F}$. Also know the statement of Theorem 20.1 (The Fundamental Theorem of Field Theory).
• Know the definition of a splitting field for a polynomial $f(x)$ over a field $\mathbb{F}$. Also know the statement of Theorem 20.2.
• Know the statement of Theorem 20.5.

Chapter 22: Finite Fields

• Know the statement of Theorem 22.1 and the definition of the Galois field of order p^n.
• Be able to construct a finite field of a given order. Also be able to construct the conversion tables for both addition and multiplication for the field you constructed.
• Know the statement of Theorem 22.3, and be able to draw the subfield lattice for a given Galois field.

Note: You are also expected to remember and be able to apply key concepts and definitions from previous exams.