Math 261
Exam 4 - Practice Problems

1. Suppose you throw a ball vertically upward. If you release the ball 7 feet above the ground at an initial speed of 48 feet per second, how high will the ball travel? (Assume gravity is $-32ft/sec^2$)

2. Find each of the following indefinite integrals:

   (a) $\displaystyle\int \frac{x^{\frac{3}{2}} - 7x^{\frac{1}{2}} + 3}{x^{\frac{1}{2}}}\,dx$
   (b) $\displaystyle\int \sin^3 x \cos x\,dx$

   (c) $\displaystyle\int 5x(x^2+1)^8\,dx$
   (d) $\displaystyle\int \frac{x}{\sqrt{x+1}}\,dx$

3. Solve the following initial value problems under the given initial conditions:

   (a) $\frac{dy}{dx} = \sin x + x^2; y = 5$ when $x = 0$
   (b) $g''(x) = 4\sin(2x) - \cos(x); g'(\frac{\pi}{2}) = 3; g(\frac{\pi}{2}) = 6$

4. Express the following in summation notation:

   (a) $2 + 5 + 10 + 17 + 26 + 37$
   (b) $x^2 + \frac{x^3}{4} + \frac{x^4}{9} + ... + \frac{x^{11}}{100}$

5. Evaluate the following sums:

   (a) $\displaystyle\sum_{2}^{5} k^2(k+1)$
   (b) $\displaystyle\sum_{3}^{20} k^3 - k^2$

6. Express the following sums in terms of n:

   (a) $\displaystyle\sum_{k=1}^{n} 3k^2 - 2k + 10$
   (b) $\displaystyle\sum_{3}^{n} k(k^2-1)$

7. Consider $f(x) = 3x^2 - 5$ in the interval $[3, 7]$

   (a) Find a summation formula that gives an estimate the definite integral of f on $[3, 7]$ using n equal width rectangles and using midpoints to give the height of each rectangle. You do not have to evaluate the sum or find the exact area.
   (b) Find the norm of the partition $P : 3 < 3.5 < 5 < 6 < 6.25 < 7$
   (c) Find the approximation of the definite integral of f on $[3, 7]$ using the Riemann sum for the partition P given in part (b).

8. Assume f is continuous on $[-5, 3]$, $\displaystyle\int_{-5}^{-1} f(x)\,dx = -7$, $\displaystyle\int_{-1}^{3} f(x)\,dx = 4$, and $\displaystyle\int_{1}^{3} f(x)\,dx = 2$. Find:

   (a) $\displaystyle\int_{3}^{-1} f(x)\,dx$
   (b) $\displaystyle\int_{-5}^{1} f(x)\,dx$

   (c) $\displaystyle\int_{-5}^{3} f(x)\,dx$
   (d) $\displaystyle\int_{-1}^{-1} f(x)\,dx$

   (e) Find the average value of f on $[-5, -1]$

9. Evaluate the following:

(a) $\displaystyle\int_1^4 x^3 + \frac{1}{\sqrt{x}} + 2\ dx$

(b) $\displaystyle\int_0^1 x^2(2x^3+1)^2\ dx$

(c) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^3(2x)\cos(2x)\ dx$

(d) $\displaystyle\int_{-\pi}^{\pi} \sin x\ dx$

(e) $\displaystyle\frac{d}{dx}\left(\int_1^3 t\sqrt{t^2-1}\ dt\right)$

(f) $\displaystyle\int_1^3 \frac{d}{dt}\left(t\sqrt{t^2-1}\right)\ dt$

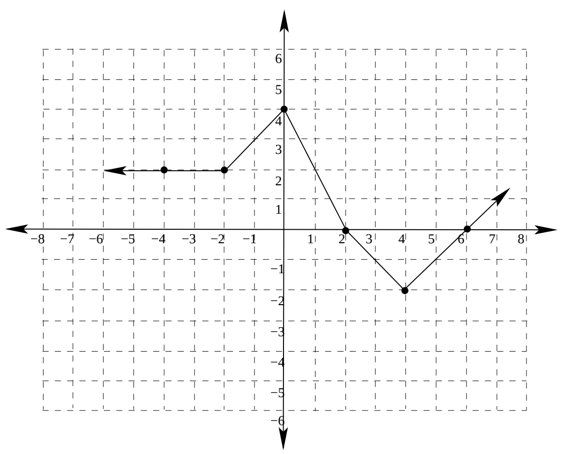
10. Suppose $G(x) = \displaystyle\int_2^x \frac{1}{t^2+1}\ dt$

(a) Find $G'(2)$

(b) Find $G'(x^2)$

(c) Find $G''(3)$

11. Given the following graph of $f(x)$ and the fact that $G(x) = \displaystyle\int_{-2}^{x} f(t)\ dt$:



(a) Find $G(6)$

(b) Find $G'(6)$

(c) Find $G''(6)$

12. Find the between the curves:

(a) $y = x^2+1$ and $y = 3x-1$

(b) $y = x^2-1$ and $y = 1-x$ on $[0,2]$

(c) $y = x$, $y = 2$, $y+x = 6$, and $y = 0$

(d) $x = y^2$, $x = 4$

13. (a) Use the Trapezoidal Rule with $n = 4$ to approximate $\displaystyle\int_0^4 2x^3\ dx$

(b) Use the Fundamental Theorem of Calculus to find $\displaystyle\int_0^4 2x^3\ dx$ exactly. How far off was your estimate?