

*Show all work for credit. Also, give exact answers unless otherwise noted.*

1. Find the derivative of each of the following functions. Simplify your answers completely.

(a)  $f(x) = \sqrt{x^2 + 1}$

(b)  $f(x) = \sin\left(\sqrt{x^2 + 1}\right)$

(c)  $f(\theta) = \frac{2 \tan \theta}{(5\theta + 1)^3}$

(d)  $f(x) = \tan(x^3)$

(e)  $f(x) = \tan^3(x)$

(f)  $f(x) = \tan^3(x^3)$

(g)  $y = (3x - 7)^3(5x^2 - 3x + 2)^5$

(h)  $f(x) = \sec(3x) \sin(3x)$

(i)  $f(x) = 3 \cos(\cot x)$

(j)  $f(x) = \frac{2x \cos(x^2)}{\sin 3x}$

2. Find the following higher order derivatives. Simplify your answers completely.

(a) Find  $f''(x)$  if  $f(x) = (x^3 - 1)^3$

(b) Find  $f''(x)$  if  $f(x) = \cos(3x) \cot(x)$

(c) Find  $f'''(x)$  if  $f(x) = \frac{4x - 3}{x + 1}$

(d) Find  $f^{(5)}(x)$  if  $f(x) = \sin 2x$

3. Assuming that each equation determines a differentiable function  $f$  such that  $y = f(x)$ , find  $f'(x)$ .

(a)  $y^2 + x^2 = 2x + 3y^2$

(b)  $2xy = x^2 - \sqrt{y}$

(c)  $x \sin y + y \sin x = 1$

(d)  $x^2(x - y)^2 = x^2 - y^2$

(e) Find  $y''$  in part (a)

(f) Find  $y''$  in part (c)

4. Find an equation for the tangent line to the graph of  $(x^2 + y^2)^2 = 50xy$  at the point  $(2, 4)$ .

5. Assume  $t^2v^3 = 1$  determines a function  $v = f(t)$ . Use implicit differentiation to show that  $v''(t) = \frac{10}{9}v^4$ .