Recall: The basic steps to solve related rates problems are:

(a) Read the problem and organize the key facts and quantities described.

(b) Draw and label a diagram that represents the situation and introduce variables for unknown quantities.

(c) Express all known facts and rates in terms of the variables introduced and their derivatives.

(d) Formulate a general equation (or equations) relating the variables.

(e) Differentiate the general equation (or equations) implicitly, yielding the general relationship between the rates.

(f) Substitute any known values and solve to find the unknown (or unknowns).

(g) State the solution (or solutions) in a complete sentence, i.e., report the problem's solution.

Solve the following related rates exercises. Show all work for credit.

1. The width of a rectangle is always half its length. At what rate is its area increasing if its width is ten centimeters and is increasing at half of a centimeter per second?

2. Suppose sand being emptied from a hopper at the rate of ten cubic feet per second forms a conical pile whose height is always twice its radius. At what rate is the radius of the pile increasing when its height is five feet?

3. A balloon two hundred feet off the ground is rising vertically at a constant rate of fifteen feet per second. An automobile passes beneath the balloon travelling along a straight road at a constant rate of forty-five miles per hour. How fast is the distance between them changing one second later?

4. Water is being collected from a block of ice with a square base. The water is produced as the ice melts in such a way that each edge of the base of the block is decreasing in length at two inches per hour, while the height of the block is decreasing at 3 inches per hour. What is the rate of the flow of water into the collecting basin when the base has an edge length of three feet and the block is three feet tall? (*You may assume that water and ice have the same density.*).

5. Suppose the observed top of a triangular object is changing. At a certain point in time, one side is ten feet long and increasing at three feet per second, a second side is fifteen feet long and decreasing at a rate of two feet per second, and the angle between these two sides is $\frac{\pi}{3}$ radians and is decreasing at a rate of $\frac{2}{25}$ radians per second. What is the rate of change of the area of the triangular region at this moment?

6. Suppose one day – even though you have a great mathematics instructor – you are so bored in calculus class that you start to watch the time pass on the clock. You begin to wonder how fast the distance between the tips of the clock hands is changing. You assume the hour hand is five inches long and the minute hand is seven inches long. How fast is the distance between the tips of the hands changing at 9:00 a.m.? **Extra Credit:** How fast is the distance between the tips of the hands changing at 10:10 a.m.?