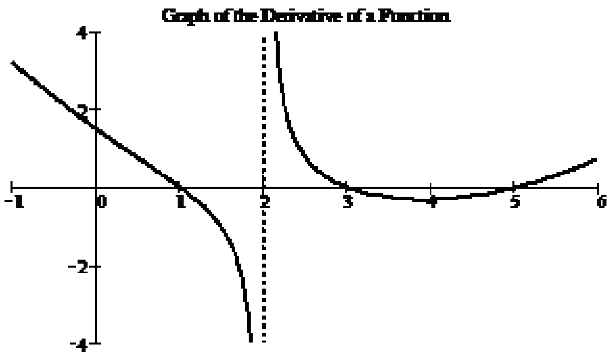
Math 261 - Lab 16 **The Second Derivative and Curve Sketching** **Name:** ______________________________

Show all work for credit.

1. Assume that the given graph is the *derivative*, f', of a continuous function f.

(a) Identify the critical numbers for f.



(b) At what value(s) of x is f'' zero?

(c) On what intervals is f concave upward? Explain.

(d) On what intervals is f concave downward? Explain.

(e) At what value(s) of x does f have a local maximum? Explain.

(f) At what value(s) of x does f have a local minimum? Explain.

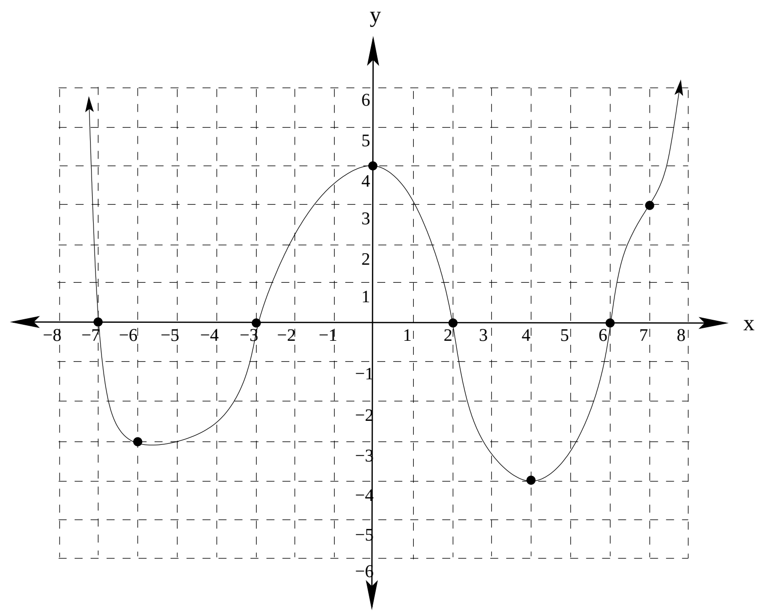
(g) Does f have a critical number at which no local extremum occurs? Explain.

(h) At what value(s) of x does f have an inflection point? Explain.

2. Given that a function g is increasing and concave down, $g(0) = 1$ and $g(10) = 7$, what can be said about the value of $g(5)$? In particular, what is the best lower bound that can be found for $g(5)$?

3. Consider the graph below. Determine the intervals where f is concave up, and concave down. Also find the x-coordinates of any inflection points.

   (a) Assume the graph shown represents f.



   (b) Assume the graph shown represents f'.

   (c) Assume the graph shown represents f''

4. Determine whether the described function has a local extrema at the indicated value. If it has an extrema, classify it.

   (a) Three is a critical number and $g''(3) = 2$.

   (b) $f'(3) = 1$ and $f''(3) = 0$.

   (c) The velocity of a particle is 0 at 8 seconds and the acceleration is negative at 8 seconds.

   (d) $R'(2) = 0$ and R changes from increasing to decreasing at 2.

5. Let $P(t)$ represent the share price of a stock a time t. Record what each of the following statements tell you about the signs of the first and second derivatives of $P(t)$.

   (a) The price of the stock is rising faster and faster.

   (b) The price of the stock is close to bottoming out.

6. On the grid provided below, sketch a graph for a function s that satisfies the following:

   Domain: $(-\infty, -3) \cup (-3, \infty)$; t-intercepts: $(-4, 0), (-1, 0)$, and $(3, 0)$; s-intercept: $(0, -3)$

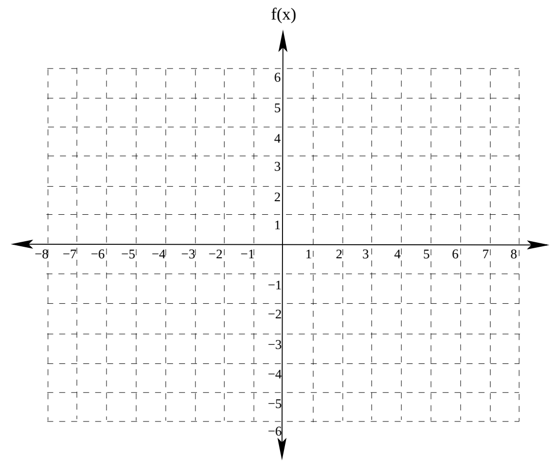
   $s'(t) > 0$ for $t \in (-\infty, -3) \cup (0, 2) \cup (5, \infty)$; $s'(t) < 0$ for $t \in (-3, 0) \cup (2, 5)$

   $s''(t) > 0$ for $t \in (-6, -3) \cup (-3, -2) \cup (3, 7)$; $s''(t) < 0$ for $t \in (-\infty, -6) \cup (-2, 0) \cup (0, 3) \cup (7, \infty)$

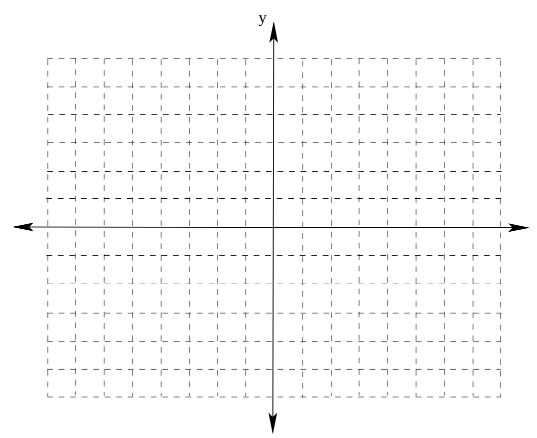
   Local Max: $(2, 1)$, Local Mins: $(0, -3)$ and $(5, -6)$;

   Inflection Points: $(-6, -2), (-2, 1), (3, 0)$, and $(7, -3)$

   $\lim_{x \to -\infty} s(t) = -\infty$; $\lim_{x \to +\infty} s(t) = -1$; $\lim_{x \to -3^-} s(t) = +\infty$; $\lim_{x \to -3^+} s(t) = +\infty$



7. For the function $g(x) = x^3 - 3x^2$, find the x-intercepts and y-intercept, any asymptotes, the intervals where it is increasing, decreasing, concave up, or concave down. Find all local extrema and inflection points.
   Then sketch the function on the grid provided.



8. For the function $f(x) = \dfrac{x}{x^2 - 4}$, find the x-intercepts and y-intercept, any asymptotes, the intervals where it is increasing, decreasing, concave up, or concave down. Find all local extrema and inflection points.
Then sketch the function on the grid provided.