Math 261 - Lab 6 **Formal and One-Sided Limits** **Name:** _____

1. Use a δ - ϵ proof to prove that: $\lim_{x \to 3} 2x - 4 = 2$.

2. Use a δ - ϵ proof to prove that: $\lim_{x \to 2} 2 - 5x = -8$.

3. For each of the following, find the limit if it exists.
If the limit does not exist, write DNE, ∞, or $-\infty$ (whichever is appropriate).

(a) $\lim_{x\to 8} 7$

(b) $\lim_{w\to 3} \dfrac{w^2-6w+9}{w^2-9}$

(c) $\lim_{m\to 5} \dfrac{\frac{1}{m}-\frac{1}{5}}{m-5}$

(d) $\lim_{h\to 0} \dfrac{(x+h)^3-x^3}{h}$

(e) $\lim_{p\to -7^+} \dfrac{\sqrt{p+7}}{p+4}$

(f) $\lim_{t\to 4^+} \left(\dfrac{t-4}{\sqrt{t^2-16}}\right)$

(g) $\lim_{x\to 1^-} \dfrac{|x^2-1|}{x^2-6x+5}$

(h) $\lim_{\theta\to 0} \dfrac{\sin^2(\theta)}{4\theta^2}$

(i) $\lim_{x\to 0} \dfrac{3x^2-9x+\sin(x)}{2x}$

(j) $\lim_{x\to 1^+} \dfrac{4x}{x^2-4x+3}$

(k) $\lim_{x\to \frac{\pi}{2}^+} \tan x$

(l) $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} -x+3 & \text{if} x<2 \\ 3 & \text{if} x=2 \\ x^2-3 & \text{if} x>2 \end{cases}$

(m) $\lim_{x\to\infty} \dfrac{5x^3-7x+1}{4x^3-8}$

(n) $\lim_{x\to\infty} \dfrac{9x^3-4x^2-5x+1}{4-x^2}$

(o) $\lim_{x\to\infty} \dfrac{2x^3-4x^2+1}{5x^5-8x}$

4. For $f(x) = \begin{cases} -x+3 & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ -x^2+6x-3 & \text{if } x > 2 \end{cases}$ and $g(x) = \begin{cases} \sqrt{x^2-2} & \text{if } x < -5 \\ 4x+5 & \text{if } -5 < x < 5 \\ x^2 & \text{if } x > 5 \end{cases}$,

find the following.

(a) $\lim_{x \to 2^+} f(x)$

(b) $\lim_{x \to 2^-} f(x)$

(c) $\lim_{x \to 2} f(x)$

(d) $f(2)$

(e) $\lim_{x \to 5^+} f(x)$

(f) $\lim_{x \to 5^-} f(x)$

(g) $\lim_{x \to 5} f(x)$

(h) $f(5)$

(i) $\lim_{x \to 2^+} g(x)$

(j) $\lim_{x \to 2^-} g(x)$

(k) $\lim_{x \to 2} g(x)$

(l) $g(2)$

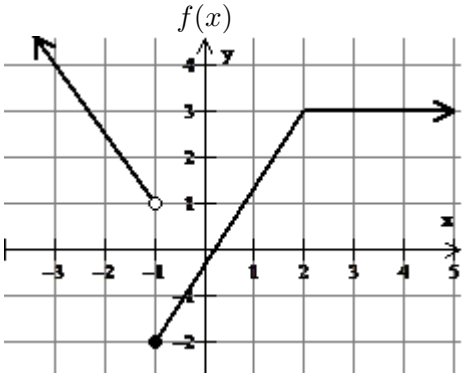
(m) $\lim_{x \to 5^+} g(x)$

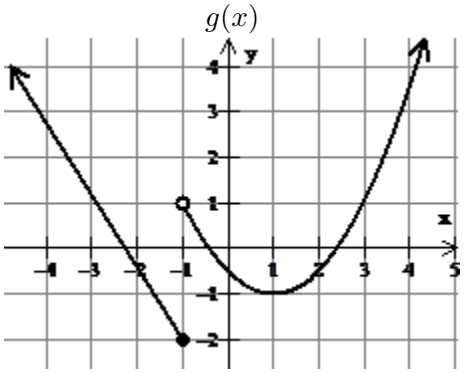
(n) $\lim_{x \to 5^-} g(x)$

(o) $\lim_{x \to 5} g(x)$

(p) $g(5)$

5. Use the following graphs to determine the value of each of the following.





(a) $\lim_{x\to -1^-} g(x) =$ (b) $\lim_{x\to -1^+} g(x) =$ (c) $\lim_{x\to -1} g(x) =$ (d) $g(-1) =$

(e) $\lim_{x\to -1^-} (f+g)(x)$ (f) $\lim_{x\to -1^+} (f+g)(x)$ (g) $\lim_{x\to -1} (f+g)(x)$ (h) $(f+g)(-1)$

(i) $\lim_{x\to 1} \dfrac{f(x)}{g(x)}$ (j) $\left(\dfrac{f}{g}\right)(1)$ (k) $\lim_{x\to -3} (f\circ g)(x)$ (l) $(f\circ g)(-3)$

(m) $\lim_{x\to 3} (f\circ g)(x)$ (n) $(f\circ g)(3)$ (o) $(g\circ f)(4)$ (p) $(f\circ g)(4) =$