Math 261
Limit Laws Handout

Example 1: How does the function $g(x) = \dfrac{x^2+2x+1}{x+1}$ behave near $x = -1$?

Question: What possible ways can a function behave as it approaches a given x-value?

Claim: If $f(x) = x$, $\lim_{x \to a} f(x) = a$. Also, if $g(x) = k$ for some constant k, then $\lim_{x \to a} g(x) = k$.

Theorem 1 (Limit Laws) If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$, then:

- $\lim_{x \to c} f(x) + g(x) = L + M$ and $\lim_{x \to c} f(x) - g(x) = L - M$.
- If k is a constant, then $\lim_{x \to c} k \cdot f(x) = k \cdot L$.
- $\lim_{x \to c} f(x) \cdot g(x) = L \cdot M$.
- $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}$, provided $M \neq 0$.
- $\lim_{x \to c} [f(x)]^n = L^n$ for any positive integer n.
- $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{L} = L^{\frac{1}{n}}$ for any positive integer n (if n is even, we need $L \geq 0$.

Theorem 2 If $P(x) = a_n x^n + a_{n-2} x^{n-1} + \cdots + a_1 x + a_0$, then $\lim_{x \to c} P(x) = P(c) = a_n c^n + a_{n-2} c^{n-1} + \cdots + a_1 c + a_0$.

Theorem 3 If $P(x)$ and $Q(x)$ are polynomials, and $Q(c) \neq 0$, then $\lim_{x \to c} \dfrac{P(x)}{Q(x)} = \dfrac{P(c)}{Q(c)}$.

Theorem 4 (The Sandwich Theorem) Suppose $g(x) \leq f(x) \leq h(x)$ for all x in some open interval containing c, except possibly at c itself. Suppose also that $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$. Then $\lim_{x \to c} f(x) = L$.