Math 143
Exam 1
9/22/2014

Name:__________________________________

Instructions: You will have 50 minutes to complete this exam. Calculators are allowed, but this is a closed book, closed notes exam. I will give credit to each problem proportional to the amount of correct work shown. Correct answers without supporting work will receive little credit. Be sure to simplify answers when possible. Also, make sure to follow directions carefully on each problem.

1. Given the points $A : (-2, 5)$ and $B : (6, -3)$:

   (a) (3 points) Find the distance between A and B

   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-2))^2 + (-3 - 5)^2} = \sqrt{(8)^2 + (-8)^2}$
   $= \sqrt{64 + 64} = \sqrt{128} = 8\sqrt{2}$.

   (b) (3 points) Find the point C such that B is the midpoint of the line segment connecting A and C

   Let $C = (x, y)$ be the point we are looking for. Then, by the midpoint formula,

   $(\frac{x-2}{2}, \frac{y+5}{2}) = B = (6, -3)$. That is, $\frac{x-2}{2} = 6$, whereby $x - 2 = 12$, so $x = 14$, and $\frac{y+5}{2} = -3$, therefore $y + 5 = -6$, so $y = -11$. Thus $C = (14, -11)$.

   (c) (3 points) Find an equation for a circle with center A and passing through the point B

   Notice that the center of the circle is $(-2, 5)$ and $r = 8\sqrt{2}$ (see part (a)).

   Then an equation for the circle is given by: $(x + 2)^2 + (y - 5)^2 = (8\sqrt{2})^2 = 128$

2. (3 points each) Suppose that the graph of $f(x)$ contains the point $(3, -1)$. Determine where this point is moved to under each of the following transformations:

   (a) $f(x + 3)$

   The point is shited 3 units left to $(0, -1)$.

   (b) $f(x) - 3$

   The point is shifted down three to $(3, -4)$

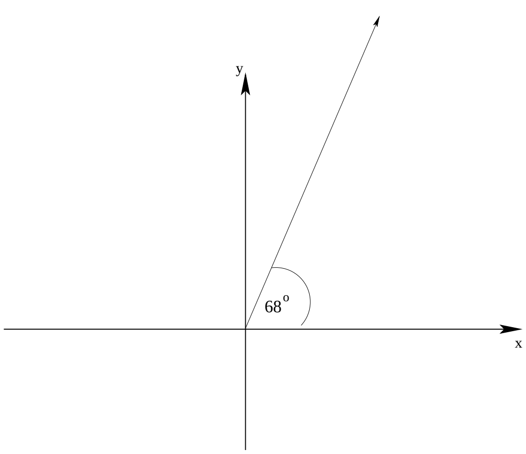
   (c) $-f(3x)$

   The point is horizontally compressed by a factor of 3 to $(1, -1)$

   It is then reflected across the x-axis, ending up at $(1, 1)$.

3. (3 points each) Given the angle $\theta = 68°$

   (a) Draw θ in standard position



   (b) Find the measure of the supplement of θ

   The supplement has measure $180° - 68° = 112°$.

   (c) Find three different angles (in degrees) that are co-terminal with θ. Exactly one of them must be negative.

   Recall that to find co-terminal angles, we add or subtract multiples of $360°$ to the measure of the original angle $68°$.

   There are many possible solutions. The most common are: $428°$, $788°$ and $-292°$.

4. (3 points each)

   (a) Let $\theta = 73.815°$. Convert the measure of θ into degree-minute-second form.

   $73.815° = 85° + .815(60)' = 85°48.9' = 85°48'.9(60)'' = 85°48'54''$

   (b) Let $\theta = 47°\,17'\,23''$. Convert the measure of θ into decimal degrees.

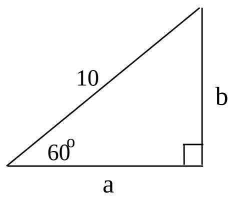
   $\theta = 47°\,17'\,23'' = \theta = 47 + \frac{17}{60} + \frac{23}{60^2}$ degrees $\approx 47.2897°$

5. (6 points) Bob has a unicycle whose wheel is two feet in diameter. Suppose that he rides it for a mile. How many revolutions does the wheel complete during the ride? Through how many degrees does a point on the wheel move? (Recall: 1 mile = 5280 feet, and $C = 2\pi r$)

   Notice that since the diameter of the wheel is 2 feet, the radius is 1 foot. Thus the circumference is 2π feet. Therefore, to find the number of revolutions, we take $\frac{5280/,/,\text{feet}}{2\pi \text{ feet per rev}} \approx 840.34$ revolutions

   From this, we can find the total number of degrees that a point on the wheel moves by multiplying: $840.34\text{revolutions} \cdot 360^{deg}\text{perrevolution} = 302,522.4°$.

6. (6 points) Find the *exact* area of the triangle below:



   Recall that for a triangle, $A = \frac{1}{2}bh$. Notice that, based on the given triangle, $\sin(60° = \frac{\sqrt{3}}{2} = \frac{b}{10}$, so $b = 5\sqrt{3}$.

   Similarly, $\cos(60° = \frac{1}{2} = \frac{a}{10}$, so $a = 5 = h$.

   Hence the exact area is $\frac{1}{2} \cdot 5\sqrt{3} \cdot 5 = \frac{25\sqrt{3}}{2}$ units squared.

7. (6 points) Fill in *exact* values in each blank in the table below:

θ (degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
240° (ref = 60°, Q3)	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
315° (ref = 45°, Q4)	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
150° (ref = 30°, Q2)	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
270° (quadrantal angle)	-1	0	undefined

8. (a) (6 points) Find the values of the six trigonometric functions for the angle θ in standard position having the point $(-5, 7)$ on its terminal side.

First, using the Pythagorean Theorem, we see that $r = \sqrt{(-5)^2 + 7^2} = \sqrt{25 + 49} = \sqrt{74}$. Note that $x = -5$ and $y = 7$.

Then we have: $\sin\theta = \frac{7}{\sqrt{74}} = \frac{7\sqrt{74}}{74}$ and $\csc\theta = \frac{\sqrt{74}}{7}$.

Similarly, $\cos\theta = \frac{-5}{\sqrt{74}} = -\frac{5\sqrt{74}}{74}$ and $\sec\theta = -\frac{\sqrt{74}}{5}$.

Finally, $\tan\theta = -\frac{7}{5}$ and $\cot\theta = -\frac{5}{7}$.

(b) (3 points) Find the measure of the angle θ from part (a) in degrees.

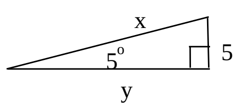
Notice that $\tan^{-1}\left(-\dfrac{7}{5}\right) \approx -54.46°$. Then the reference angle $\theta_R \approx 54.46°$. From this, since θ is in the second quadrant, we must have $\theta \approx 180° - 54.46° = 125.54°$.

(c) (3 points) Express the direction of the ray from the origin through the point $(-5, 7)$ as a bearing.

To find the bearing using method 1, we measure clockwise from North. Then the bearing is: $270° + 54.46° = 324.26°$.

Using method 2, we note that $90° - 54.46° = 35.54°$. Then the bearing is $N35.54°W$.

9. (6 points) Given that $\sin\theta = \frac{6}{11}$ and $\cos\theta < 0$, find the exact value of both $\cos\theta$ and $\tan\theta$.

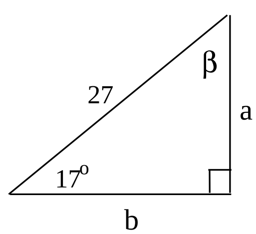


Using the triangle above, $x = 11^2 - 6^2 = 121 - 36 = 85$. Also, since $\sin\theta > 0$ and $\cos\theta < 0$, θ is in the second quadrant, so $\tan\theta < 0$

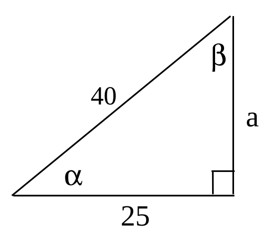
Therefore, $\cos\theta = -\frac{\sqrt{85}}{11}$ and $\tan\theta = -\frac{6}{\sqrt{85}}$

10. (6 points each) Given the indicated parts of the triangle $\triangle ABC$, find all remaining parts. Estimate your answers to within 2 decimal places.

(a)



(b)

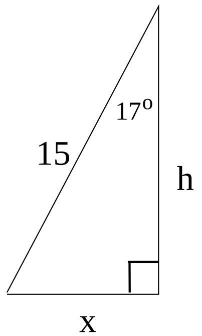


For part (a), First, since the angles of a triangle total 180°, $\beta = 90° - \alpha = 90° - 17° = 73°$. Next, notice that $\sin 17° = \frac{a}{27}$, so $a = 27\sin 17° \approx 7.89$. Similarly, $\cos 17° = \frac{b}{27}$, so $b = 27\cos 17° \approx 25.82$.

For part (b), by the Pythagorean Thoerem, $40^2 = a^2 + 25^2$, so $a = \sqrt{40^2 - 25^2} \approx 31.22$. Next, $\cos\alpha = \frac{25}{40}$, therefore, $\alpha = \cos^{-1}(\frac{25}{40}) \approx 51.32°$. Finally, $\beta = 90 - \alpha \approx 90° - 51.32° = 38.68°$

11. (12 points) A 15 foot extension ladder is leaning against a wall forming an angle of 17°.

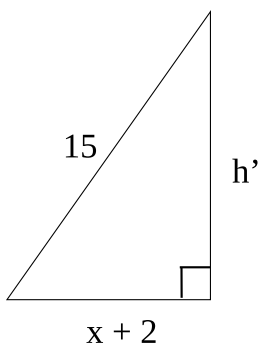
(a) How far is the base of the ladder from the wall?



Based on the triangle constructed above, $\sin 17° = \frac{x}{15}$. Therefore, $x = 15 \sin 17° \approx 4.39$ feet.

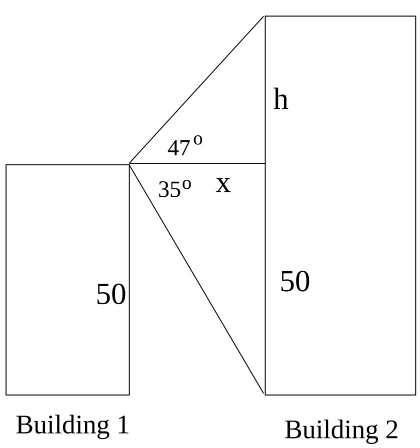
(b) If the distance between the base of the ladder and the wall is increased by 2 feet, how much does the top of the ladder move down the wall?

First notice that based on the triangle above, since $\cos 17° = \frac{h}{15}$, the original height of the ladder was $h = 15 \cos 17° \approx 14.34$. To find the new height, we build a triangle for the ladder after its base has been moved 2 feet further from the wall:



Based on the new triangle, using the Pythagorean Theorem, we see that $h' = \sqrt{15^2 - (x+2)^2} = \sqrt{15^2 - (6.39)^2} \approx 13.57$ feet. Therefore, the top of the ladder moved aproximately $14.34 - 13.57 = .77$ feet down the wall.

12. (12 points) Suppose you are standing on the roof of a 50 foot tall building. From your position on the roof, you have a clear view of a taller building across the street. You measure the angle of elevation to the top of the other building and find that it is 47°, while the angle of depression to the base of the other building is 35°. Find the height of the second building.



The diagram above shows the situation described in this problem. To find the total height of the second building, we first find the distance between the two buildings using the fact that $\tan 35° = \frac{x}{50}$, therefore, $x = \frac{50}{\tan 35°} \approx 71.41$ feet.

Next, we notice that $\tan 47° = \frac{h}{x}$, so $h \approx 71.41 \tan 47° \approx 76.6$ feet.

Thus, the total height of the building is approximately $50 + 76.6 = 126.6$ feet.