Math 143
Exam 2
10/17/2014

Name:______________________________

Instructions: You will have 50 minutes to complete this exam. Calculators are allowed, but this is a closed book, closed notes exam. I will give credit to each problem proportional to the amount of correct work shown. Correct answers without supporting work will receive little credit. Be sure to simplify answers when possible. Also, make sure to follow directions carefully on each problem.

1. (4 points each)

   (a) Convert 210° into radian measure. Leave your answer as a multiple of π.

   $210° \cdot \frac{\pi \text{rad.}}{180°} = \frac{21\pi}{18} = \frac{7\pi}{6}$ radians.

   (b) Convert $\frac{5\pi}{12}$ into degree measure.

   $\frac{5\pi}{12} \cdot \frac{180°}{\pi \text{rad.}} = \frac{5 \cdot 180}{12} = \frac{5 \cdot 30}{2} = 5 \cdot 15 = 75°.$

2. (4 points each) Find the exact value of the following:

   (a) $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

   (b) $\cos(\frac{7\pi}{4}) = \frac{\sqrt{2}}{2}$

   Note that θ is in Q4 and $\theta_R = \frac{\pi}{4}$.

   (c) $\tan(\frac{7\pi}{6}) = \frac{\sqrt{3}}{3}$

   Note that θ is in Q3 and $\theta_R = \frac{\pi}{6}$.

   (d) $\sec(\pi) = -1$

3. Consider a wheel of radius 7cm.

   (a) (5 points) Suppose the wheel is rotated through an angle of 135°. Find the total distance traveled by a point on the outside edge of the wheel.

   Recall that $s = r\theta$ given the arc length for an angle θ in radians and a circle of radius r.

   Therefore, $s = 135° \cdot \frac{\pi}{180} \cdot 7 = \frac{5 \cdot 27 \cdot 7 \cdot \pi}{36 \cdot 5} = \frac{3 \cdot 7 \cdot \pi}{4} = \frac{21\pi}{4}$cm. ≈ 16.49 cm.

   (b) (8 points) Suppose the 7 cm wheel has been attached to a conveyor belt and is rotating at 100 revolutions per minute. Find the angular speed of the wheel in radians per second. Then find the linear speed of the belt in centimeters per second.

   Finding the angular speed is a units conversion problem: $\frac{100\text{rev}}{\text{min}} \cdot \frac{1\text{min}}{60\text{sec}} \cdot \frac{2\pi \text{rad.}}{1\text{rev}} = \frac{10\pi}{3}$ rad/sec ≈ 10.472 rad/sec.

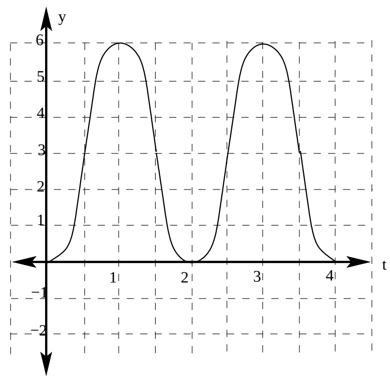
   To find the linear linear speed, we simply multiply by the radius: $\frac{10\pi}{3} \cdot 7 = \frac{70\pi}{3}$ cm/sec ≈ 73.3 cm/sec.

4. (6 points) Find all angles s in $[0, 2\pi)$ such that $\cos s = .725$. You should approximate your answers to three decimal places.

   First notice that we expect to find two values, one in the first quadrant and another in the 4th quadrant.

   To find the value in the first quadrant, we use the inverse cosine function (making sure to compute in radian mode): $cos^{-1}(.725) \approx 0.75976$ radians.

   To find the value in the fourth quadrant, we subtract this value from 2π: $2\pi - 0.75976 \approx 5.5234$ radians.



5. Consider the graph:

   (a) (6 points) Find the amplitude, period, and midline for the graph.

   Notice that the max value is 6 and the min value is zero. Therefore, the amplitude is $\frac{6-0}{2} = 3$, and the midline is given by $y = \frac{6+0}{2} = 3$, or $y = 3$.

   Measuring from min to min or from max to max, we see that the period is 2 units.

   (b) (6 points) Express the function shown with an equation of the form: $y = a\sin(bt + c) + d$

   Since a sine graph normally starts at the midline and proceeds to a maximum, to express this graph as a sine function, we must introduce a phase shift by $\frac{1}{2}$ units right.

   Also, since the period is 2, then $\frac{2\pi}{b} = 2$, so we must have $b = \pi$.

   Then this graph is described by the equation: $y = 3\sin(\pi(t - \frac{1}{2})) + 3$

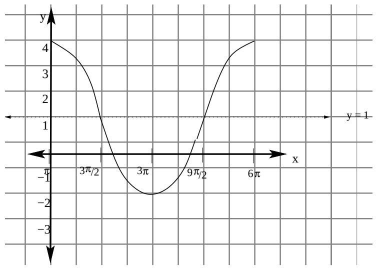
   (c) (4 points) Express the function shown with an equation of the form: $y = a\cos(bt + c) + d$
   Note that the amplitude, midline, and period do not change. To express this as a cosine graph, we have two primary options, we can introduce a phase shift of 2 unit right, or we can reflect the graph about the x-axis.

   Then either $y = 3\cos(\pi(t - 1)) + 3$ or $y = -3\cos(\pi t) + 3$

6. For each function below, find the amplitude, period, phase shift, and midline of the function, and then carefully draw one period the graph of the function, clearly labeling key points.

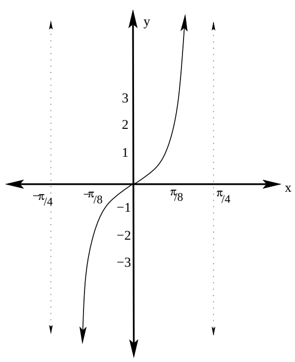
(a) (10 points) $y = 3\cos\left(\frac{1}{3}x\right) + 1$

Notice that the amplitude is 3. Since $b = \frac{1}{3}$, the period is $\frac{2\pi}{\frac{1}{3}} = 6\pi$. We also see that the midline is $y = 1$, and there is no phase shift. The increment is: $\frac{6\pi}{4} = \frac{3\pi}{2}$. The result is the graph below:



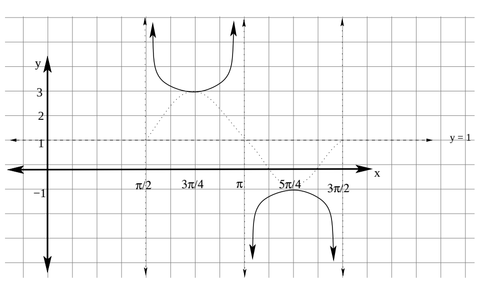
(b) (10 points) $y = 3\tan(2x)$

Notice that the graph is stretched vertically by a factor of 3. Since $b = 2$, and the standard period of tan is π, the period is $\frac{\pi}{2}$. We also see that the midline is $y = 0$, and there is no phase shift. The increment is $\frac{\pi}{8}$. The resulting graph is:



(c) (10 points) $y = 2\csc(2x - \pi) + 1$

We will graph this by first considering the related sine graph, $y = 2\sin(2x - \pi)$. Notice that the amplitude is 2. Since $b = 2$, the period is $\frac{2\pi}{2} = \pi$. We also see that the midline is $y = 1$, and, factoring the argument give $2(x - \frac{\pi}{2})$, so the phase shift is $\frac{\pi}{2}$ units right. To see where the graph begins and ends and doublecheck the period, e solve the inequality: $0 \leq 2x - \pi \leq 2\pi$, which gives $\pi \leq 2x \leq 3\pi$, or $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$. The increment is $\frac{\pi}{2}$. The result is the graph below:



7. Suppose that a weight is attached to one end of a spring. The other end of the spring has been securely clamped to a stand. An experimenter pulls down on the weight and then releases it. She then uses a video recording device, along with accurate markings on the wall behind the stand, to record the height of the bottom of the weight at various times. The table below summarizes her data:

Time in seconds:	0	.25	.5	.75	1.0	1.25	1.5	1.75	2.0	2.25	2.5
Height in cm:	85.0	91.85	105.55	112.4	105.55	91.85	85	91.85	105.55	112.4	105.55

(a) (4 points) Find both the average height of the spring and the amplitude of the spring.

Notice that the max value is 112.4 and the min value is 85.0. Therefore, the amplitude is $\frac{112.4-85.0}{2} = 13.7$ cm. The average value is given by $y = \frac{112.4+85.0}{2} = 98.7$ cm. (not the arithmetic mean!)

(b) (4 points) Find the period of a function describing the motion of this spring.

Measuring from min to min or from max to max, we see that the period is 1.5 seconds.

(c) (6 points) Write the equation of a trigonometric function that describes the height of this spring as a function of time in seconds.

Since the period is 1.5, then $\frac{2\pi}{b} = \frac{3}{2}$, so $b = 2\pi \cdot \frac{2}{3} = \frac{4\pi}{3}$. Since the graph begins at a minimum, we must either use a phase shift or a cosine graph with a x-axis reflection. We choose the latter since it is a bit simpler:

$y = -13.7\cos\left(\frac{4\pi}{3}t\right) + 98.7$