Math 143
Trigonometric Function Definitions

Trigonometric Functions

Definition: Let (x, y) be a point other than the origin on the terminal side on an angle θ in standard position. The distance from the point to the origin is $r = \sqrt{x^2 + y^2}$. The six trigonometric functions are defined as follows.

$\sin\theta = \dfrac{y}{r}$ $\qquad$ $\csc\theta = \dfrac{r}{y}$ if $y \neq 0$

$\cos\theta = \dfrac{x}{r}$ $\qquad$ $\sec\theta = \dfrac{r}{x}$ if $x \neq 0$

$\tan\theta = \dfrac{y}{x}$ if $x \neq 0$ $\quad$ $\cot\theta = \dfrac{x}{y}$ if $y \neq 0$

Recall: Quadrantal Angles are angles which have measure equal to a multiple of 90°. The following table gives the values of the main quadrantal angles.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
0°	0	1	0	undefined	1	undefined
90°	1	0	undefined	0	undefined	1
180°	0	-1	0	undefined	-1	undefined
270°	-1	0	undefined	0	undefined	-1
360°	0	1	0	undefined	1	undefined

Reciprocal Identities:

$\sin\theta = \dfrac{1}{\csc\theta}$ $\qquad$ $\csc\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ $\qquad$ $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$ $\qquad$ $\cot\theta = \dfrac{1}{\tan\theta}$

Signs of Trigonometric Functions by Quadrant:

θ in Quadrant	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
I	+	+	+	+	+	+
II	+	-	-	-	-	+
III	-	-	+	+	-	-
IV	-	+	-	-	+	-

Ranges of Trigonometric Functions:

Function	**Range**
$\sin\theta$, $\cos\theta$	$[-1, 1]$
$\tan\theta$, $\cot\theta$	$(-\infty, \infty)$
$\sec\theta$, $\csc\theta$	$(-\infty, -1] \cup [1, \infty)$

Pythagorean Identities:

$\sin^{\theta} + \cos^2\theta = 1$ $\quad$ $\tan^2\theta + 1 = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

Quotient Identities:

$\tan\theta = \dfrac{sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Right Triangle Side Versions of Trigonometric Functions:

$\sin\theta = \frac{O}{H} = \frac{1}{\csc\theta}$	$\cos\theta = \frac{A}{H} = \frac{1}{\sec\theta}$	$\tan\theta = \frac{O}{A} = \frac{1}{\cot\theta}$
$\csc\theta = \frac{H}{O} = \frac{1}{\sin\theta}$	$\sec\theta = \frac{H}{A} = \frac{1}{\cos\theta}$	$\cot\theta = \frac{A}{O} = \frac{1}{\tan\theta}$

III. Special Values of Trigonometric Functions:

θ (radians)	θ (degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$