Math 261
Antidifferentiation Handout

Definition: A function $F(x)$ is an **antiderivative** of a function $f(x)$ on an interval I if $F'(x) = f(x)$ for all $x \in I$.

Examples: Find an antiderivative for each of the following functions:

1. $f(x) = 2$
2. $f(x) = 2x$
3. $f(x) = x^2$
4. $f(x) = \cos x$
5. $f(x) = \sin x$
6. $f(x) = \sec^2 x$
7. $f(x) = \frac{1}{2\sqrt{x}}$
8. $f(x) = (2x+1)^2$

Theorem 6: If F is an antiderivative of f on an interval I, then the most general antiderivative of f is $F(x) + C$, where C is an arbitrary constant.

Proof:
Let $F(x)$ be an antiderivative of a function f on an interval I. Then, by definition, $F'(x) = f(x)$ for all $x \in I$. Consider $F(x) + C$ for some constant C.

Differentiating, $\frac{d}{dx}(F(x) + C) = \frac{d}{dx}(F(x)) + \frac{d}{dx}(C) = f(x) + 0 = f(x)$ for each $x \in I$. Hence $F(x) + C$ is an antiderivative of f for any constant C.

Basic Antidifferentiation Formulas: Let n be an integer and r a non-zero real number constant.

1. If $f(x) = r$, $F(x) = rx + C$
2. If $f(x) = x^n$ for $n \neq -1$, $F(x) = \frac{1}{n+1}x^{n+1} + C$
3. If $f(x) = \sin(rx)$, $F(x) = -\frac{1}{r}\cos(rx) + C$
4. If $f(x) = \cos(rx)$, $F(x) = \frac{1}{r}\sin(rx) + C$
5. If $f(x) = \sec^2(rx)$, $F(x) = \frac{1}{r}\tan(rx) + C$
6. If $f(x) = \csc^2(rx)$, $F(x) = -\frac{1}{r}\cot(rx) + C$
7. If $f(x) = \sec(rx)\tan(rx)$, $F(x) = \frac{1}{r}\sec(rx) + C$
8. If $f(x) = \csc(rx)\cot(rx)$, $F(x) = -\frac{1}{r}\csc(rx) + C$

   Suppose $F(x)$ is an antiderivative of $f(x)$ and $G(x)$ is an antiderivative of $g(x)$.

9. If $rF(x) + C$ is an antiderivative of $r \cdot f(x)$. In particular, $-F(x) + C$ is an antiderivative of $-f(x)$.
10. $F(x) \pm G(x) + C$ is an antiderivative of $f(x) \pm g(x)$.