Math 261

Tangent Lines and the Derivative Handout

• Recall that a *secant line* meets the graph of a function at two points. The slope of the secant line through the points $P(a, f(a))$ and $Q(a+h, f(a+h))$ is equal to the *average rate of change* of the function f over the interval $[a, a+h]$.

For example, the slope of a secant line to a position (displacement) graph is the *average speed* of the object being described over the time period between the two points on the graph intersecting the line.

• Recall that a *tangent line* meets the graph of a function at a point $P(a, f(a))$, and has slope equal to the *instantaneous rate of change* of the function f at the point P.

For example, the slope of a tangent line to a position (displacement) graph is the *instantaneous velocity* of the object being described at that particular point in time.

• The **slope of the curve** $y = f(x)$ at the point $P(a, f(a))$ (the slope of the tangent line to curve at this point) is given by:

$\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$, provided this limit exists.

• The **tangent line** to the curve at P is the line through P with this slope.

Example: If $f(x) = -2x^2 + 8x$, then the slope of this curve at the point $P(a, f(a))$ is:

$$m_a = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{-2(a+h)^2 + 8(a+h) - (-2a^2 + 8a)}{h} = \lim_{h \to 0} \frac{-2(a^2 + 2ah + h^2) + 8a + 8h + 2a^2 - 8a}{h} =$$
$$\lim_{h \to 0} \frac{-2a^2 - 4ah - 2h^2 + 8a + 8h + 2a^2 - 8a}{h} = \lim_{h \to 0} \frac{-4ah - 2h^2 + 8h}{h} = \lim_{h \to 0} -4a - 2h + 8 = -4a + 8$$

In particular, if $a = 1$, then $m_a = -4 + 8 = 4$

• To find an equation for a tangent line to a function f at a point $P(a, f(a))$, we use the point P together with the slope m_a.

Example: given the function $f(x) = -2x^2 + 8x$, and $x = 1$, $(1, f(1)) = (1, 6)$, and $m = 4$, so the equation of the tangent line at this point is given by: $y - 6 = 4(x - 1)$, or $y = 4x + 2$.

Definition: The **derivative of a function f at a point** x_0, denoted $f'(x)$, is:

$f'(x_0) = \lim_{h \to 0} \dfrac{f(x_0 + h) - f(x_0)}{h}$, provided this limit exists.