Math 261
Exam 2 - Practice Problems

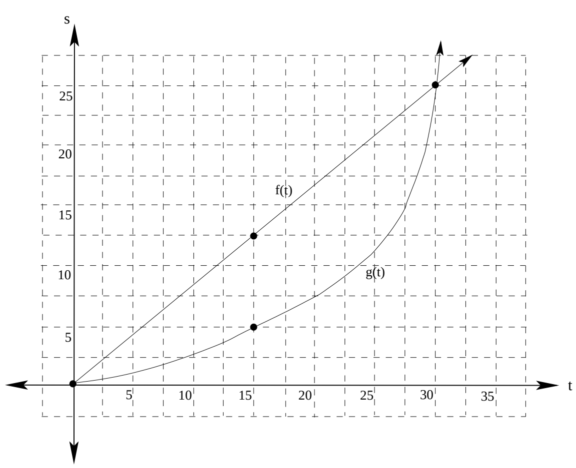
1. Find the derivative $y' = \frac{dy}{dx}$ for each of the following:

   (a) $y = \pi^2 x + \pi x^2$
   (b) $y = \cot x$
   (c) $y = \sqrt{x}\sec(x^2)$
   (d) $y = 2\tan^3(2x^3)$
   (e) $y = \dfrac{x^2 - 7\cos(3x)}{x + \sin(3 - 2x)}$
   (f) $x^2 y + 3xy - 5y^2 = 7$
   (g) $\cos^2(xy) = 1$

2. Use the formal limit definition of the derivative to find the derivative of the following:

   (a) $f(x) = x^2 - 3x$
   (b) $f(x) = \dfrac{2}{x - 3}$
   (c) $f(x) = \sqrt{x - 2}$

3. The position of two cars, car A and car B, both starting side by side on a straight road, is given by $f(t)$ and $g(t)$, where $f(t)$ is the distance traveled car A in feet, and $g(t)$ is the distance traveled car B in feet, and t is in minutes (see the graph below):



   (a) How fast is car A going at time $t = 15$?
   (b) Find the average rate of change of car B on the time interval $[0, 15]$.
   (c) Which car is traveling faster at time $t = 15$?
   (d) Which car is traveling faster at time $t = 30$?
   (e) What can you say about the relative positions of the two cars at time $t = 30$?

4. Use the quotient rule to derive the formula for the derivative of $\tan(x)$.

5. Use the product rule to prove that $D_x[f(x)g(x)h(x)] = f'(x)g(x)h(x)+f(x)g'(x)h(x)+f(x)g(x)h'(x)$

6. If $f(x) = \sqrt{3x-5}$, find the intervals where $f(x)$ is continuous, and find the intervals where $f(x)$ is differentiable.

7. If $f(x) = 3x^4 - 5x^2 + 7x - 12$, use differentials to approximate $f(1.1)$

8. Use differentials to approximate $\sqrt{1.2}$. How good is your approximation?

9. Use differentials to estimate $\sqrt[3]{9}$. How good is your approximation?

10. Suppose helium is being pumped into a spherical balloon at a rate of 4 cubic feet per minute. Find the rate at which the radius is changing when the radius is 2 feet.

11. Dr. Von Klausen has just invented a shrink ray and decides to try it out on a test object: a cylinder whose height is twice its radius. The shrink ray has been calibrated so that the proportions of the cylinder remain the same throughout the test. A few seconds into the test, the radius of the cylinder is decreasing at 2 inches per second, and the height is 4 inches. At what rate is the volume of the cylinder changing at that time? (Be sure to include units in your answer)

12. Find the equation of the tangent line to the graph of $f(x) = \tan(4x)$ when $x = \frac{3\pi}{16}$

13. Find the equation of the tangent line to the graph of $y = \sec(2x)$ when $x = \frac{\pi}{6}$.

14. Find the points on the graph of $y = 2x^3 + 3x^2 - 72x + 5$ at which the tangent line is horizontal.

15. Find the equation of the tangent line to the graph of the relation $x^2y + 3y^2 = 3x - 7$ at the point $(2, -1)$

16. Draw the graph of a function $f(x)$ that is continuous when $x = 3$, but is not differentiable when $x = 3$.

17. Find $g'(2)$ if $h(x) = f(g(x))$, $f(3) = -2$, $g(2) = 3$, $f'(3) = 5$, and $h'(2) = -30$.

18. Given that $f(2) = -3$, $g(2) = 2$, $f'(2) = \frac{1}{2}$, $g'(2) = -5$, and $h(x) = f(g(x))$.

    Find the following:

    (a) $(f - g)'(2)$ (b) $(fg)'(2)$

    (c) $\left(\frac{f}{g}\right)'(2)$ (b) $h'(2)$

19. Find $f^{(8)}(x)$ if $f(x) = \sin(2x)$

20. Find $f^{(13)}(x)$ if $f(x) = x^{12} + 7x^5 - 3x^3 - 1$

21. Use linearization to find a good approximation of $\sqrt[3]{10}$.

22. A company manufactures wooden cubes. Each side of the finished cubes are 5 inches long, with a maximum error of $\pm.2$ inches per side. Use differentials to estimate the maximum error in the volume of the cube. Then, compare your estimate with the error in volume of a cube with largest possible volume manufactured within the given error tolerances.