Math 261

Exam 4 Review Sheet

Section 5.1 Area and Estimating with Finite Sums

• Know how to estimate the area between a function f(x) and the x-axis over an interval [a, b] using rectangular approximations such as: upper sums, lower sums, left hand sums, right hand sums, and midpoint sums based on equally sized subintervals or unequally sized subintervals.

- Understand that these approximations become more accurate as more rectangles are used.
- Be able to solve application problems involving displacement or total distance traveled.

Section 5.2 Sigma Notation and Limits of Finite Sums

• Understand summation notation and the basic properties of sums. Be able to apply them to find the value of a specific sum.

- Be able to find a summation notation representation for a given sequence of numbers.
- Memorize the summation formulas: $\sum_{k=1}^{n} c = nc$, $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, and $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$
- Be able to apply these summation formulas to find the value of a given sum either as a number or as an expression in n.
- Be able to use a finite sum to **approximate** the area under a continuous function on an interval [a, b].
- Be able to take the limit of a finite sum in order to find the exact area under a continuous function on an interval [a, b].

Section 5.3 The Definite Integral

- Understand the definition of a partition P of a closed interval [a, b] and the definition of the norm of a partition.
- Understand how to find the Riemann sum associated with a particular partition.

• Know the definition of the definite integral of a function f on an interval [a, b] (i.e. the limit of Riemann sums on the interval as the norm $||P|| \rightarrow 0$, provided the limit exists).

- Understand the connection between the definite integral and area between a function and the x-axis.
- Know that continuous functions are integrable, but that not all functions are integrable on a given interval [a, b].
- Know the basic properties of definite integrals and be able to apply them to carry out computations (see pg. 266 in your textbook).
- Be able to find the average value of a function on a given interval.

Section 5.4 The Fundamental Theorem of Calculus

- Memorize the statement of the Mean Value Theorem for definite integrals.
- Memorize both parts of the statement of the fundamental theorem of calculus.
- Be able to apply the fundamental theorem of calculus to compute definite integrals of continuous functions.
- Understand the relationship between differentiation, integration, and the first part of the Fundamental Theorem and be able to perform related computations.
- Understand the meaning of the definite integral of a rate of change and be able to solve related application problems.

• Understand the difference between signed area and unsigned area and be able to decide which is appropriate to use in a given situation.

Section 5.5, 5.6a Indefinite Integrals and the Substitution Method

- Understand the method of substitution and be able to use it to solve indefinite integrals.
- Be able to use the method of substitution to compute the value of a definite integral.
- Be able to use trigonometric identities to find antiderivatives involving $\sin^2(u)$ and $\cos^2(u)$.
- Be able to use substitution to solve initial value problems.

Section 5.6b Area Between Curves

- Be able to find the Area between a function and the x-axis using the Fundamental Theorem of Calculus.
- Be able to find the area of a region enclosed between two functions f(x) and g(x) with f(x) > g(x) (perhaps on an given interval) by setting up a definite integral and applying the Fundamental Theorem of Calculus.

• Be able to find the area of a region enclosed between two functions f(y) and g(y) with f(y) > g(y) (perhaps on an given interval) by setting up a definite integral and applying the Fundamental Theorem of Calculus.

• Be able to find the area of a region bounded by more than two functions or a region bounded by functions that cross each other one or more times.

Section 5.7 Numerical Integration

• Know how to estimate a definite integral using the Trapezoid Rule.