Math 261

The Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus Part 1:

Let f be a continuous function on the interval $[a, b]$, then $F(x) = \int_a^x f(t)\ dt$ for every x in $[a, b]$ is continuous on $[a, b]$ and differentiable on (a, b) and its derivative is $f(x)$. That is, $F'(x) = \dfrac{d}{dx}\int_a^x f(t)\ dt = f(x)$.

Examples:

1. If $y = \int_a^x \sin(3t)\ dt$, then $\frac{dy}{dx} = \sin(3x)$.
2. If $y = \int_x^2 (5t^2 + 7)^8\ dt$, then $\frac{dy}{dx} = \frac{d}{dx}\int_x^2 (5t^2 + 7)^8\ dt = -\frac{d}{dx}\int_2^x (5t^2 + 7)^8\ dt = -(5x^2 + 7)^8$.
3. If $y = \int_1^{x^4} \tan(t)\ dt$, we set $u = x^4$. Then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(\int_1^u \tan(t)\ dt\right) \cdot \frac{du}{dx} = (\tan(u)) \cdot 4x^3 = 4x^3 \tan(x^4)$.

The Fundamental Theorem of Calculus Part 2:

If f is continuous at every point in $[a, b]$ and F is any antiderivative of f on $[a, b]$, then $\int_a^b f(x)\ dx = F(b) - F(a)$

Examples:

1. $\int_a^b x\ dx = \frac{1}{2}x^2\ \Big|_a^b = \frac{b^2}{2} - \frac{a^2}{2}$
2. $\int_1^3 3x^2\ dx = x^3\ \Big|_1^3 = 3^3 - 1^3 = 27 - 1 = 26$
3. $\int_0^4 \sqrt{2t+1}\ dt = \frac{1}{3}(2t+1)^{\frac{3}{2}}\ \Big|_0^4 = \frac{1}{3}(9)^{\frac{3}{2}} - \frac{1}{3}()^{1\frac{3}{2}} = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$

The Net Change Theorem: The net change in a function $F(x)$ over an interval $a \le x \le b$ is the integral of its rate of change:

$F(b) - F(a) = \int_a^b F'(x)\ dx$.

More Examples: