Math 261 Section 02
Final Exam Review Sheet

Final Exam: Wednesday, December 17th from 11:30am-1:30pm in Bridges Room 264

Review Material

• Piecewise defined functions, graphs of functions, finding the domain and range of a function, key values of trig functions, solving trig equations.

Limits and Continuity

• Be able to investigate limits by constructing a table of values.
• Be able to evaluate limits using algebra, using graphs, and using the properties of limits.
• Know how to find one - sided limits and limits of piece-wise defined functions.
• **Memorize** the formal ϵ-δ definition of a limit.
• Given a function and a specific ϵ value, be able to find the related δ value.
• Know how to prove the value of a limit using the formal definition of a limit.
• Understand limits as $x \to \pm\infty$ and one and two sided limits where the function value approaches to $\pm\infty$
• Know the definition of continuity of a function at a point $x = c$
• Be able to classify points of discontinuity as removable, jump, or infinite discontinuities.
• Understand and be able to apply the definition of continuity on open and closed intervals: (a, b) and $[a, b]$
• Know results about the limits of polynomial, rational functions, roots of functions and combinations of functions.
• Be able to find the points of discontinuity of a given function.
• Know the statement and basic applications of the Intermediate Value Theorem.

Tangent Lines, Rates of Change, and the Derivative

• Understand secant lines, finding the average rate of change over an interval, and finding the instantaneous rate of change at a point.
• **Memorize** the formal limit definition of the derivative of a function and be able to use it to find the derivative of a function.
• Know how to use the derivative to find the equation of a tangent line to a function.
• Understand the definition of differentiability on open and closed intervals and be able to find where a function is differentiable.
• Understand right and left hand derivatives, vertical tangent lines and cusps.
• Remember that differentiable functions are continuous, but continuous functions may not be differentiable.
• Memorize the differentiation formulas for constant functions, power functions, the 6 trig functions, differentiation rules for sums, differences, products, and quotients of functions, and the chain rule and be able to apply these rules to find the derivatives of various functions.
• Be able to solve application problems involving tangent lines and rates of change and proofs of trigonometric differentiation formulas using the quotient rule.

Applications of the Derivative

• Know how to find the derivative of an implicit function and how to find equations of tangent lines to points on implicit curves.
• Know how to use implicit differentiation to solve *related rates problems.*
• Know the definitions of increasing, decreasing, constant, maximum value, minimum value, local maximum, and local minimum and be able to apply them to the graph of a function.
• Understand the connection between critical numbers and local extrema and be able to find the critical numbers of a given function.
• Know the statement of the Extreme Value Theorem and be able to apply it to find the extrema of a continuous function on a closed interval.
• **Memorize** the statements of the Mean Value Theorem (MVT) and the Intermediate Value Theorem (IVT) and be able to determine whether a given function satisfies the hypotheses of the MVT and/or the IVT.
• Be able to apply the MVT and/or IVT to reach conclusions about a function given via a formula or a table of values.
• Understand the connection between the sign of the derivative and the increasing/decreasing behavior of a function.
• Know how to find the intervals where a function is increasing/decreasing and classify its local extrema by analyzing the first derivative of the function or by using the second derivative test.
• Understand the connection between the concavity of a function and the sign of the second derivative of the function and know how to find the intervals where a function is concave up/down and find any inflection points by analyzing the second derivative of the function.
• Be able to reach conclusions about the shape of the graph of a function by looking at the graph of its first or second derivative.

• Understand how to find the vertical and horizontal asymptotes of a function as well as the intercepts of a function.
• Be able to analyze the first and second derivatives of a function in order to find the intervals where it is increasing/decreasing, concave up/down, and to find the coordinates of all local extrema and inflection points.
• Be able to use limits to find the vertical and horizontal asymptotes of a function
• Be able to combine information about asymptotes, intercepts, local extrema, inflection points, increasing/decreasing intervals, and concavity in order to draw an accurate graph of a function.
• Understand the idea of *optimization* and how to use information from the first and second derivative in order to find maximum and minimum values for a given function under certain constraints and be able to apply these methods to specific situations.
• Understand the relationship between position, velocity, and acceleration and be able to find equations for position, velocity, and acceleration satisfying given initial conditions.
• Understand how to use Newton's method to approximate a zero of a function and be able to use Newton's method to approximate nth roots of numbers.

Antiderivatives and Indefinite Integrals and Definite Integrals

• Know the definition of an antiderivative of a function and how to find the indefinite integral of a function, including the arbitrary constant of integration.
• Know how to solve initial value problems by using initial conditions to solve for constants of integration.
• Know how to carry out a change of variables for definite and indefinite integrals.
• Understand summation notation and the basic properties of sums.
• Memorize the summation formulas: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, and $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$ and be able to apply them to find the values of given sums.
• Understand how to use rectangles to approximate the area under a continuous function on an interval $[a, b]$ using left hand, right hand, and midpoint heights.
• Understand how to use sums and limits to find the exact area under a continuous function on an interval $[a, b]$.
• Understand the definition of a partition P of a closed interval $[a, b]$ and the definition of the norm of a partition.
• Understand how to find the Riemann sum area approximation associated with a particular partition.
• Know the definition of the definite integral of a function f on an interval $[a, b]$ (i.e. the limit of Riemann sums on the interval as the norm $\|P\| \to 0$, provided the limit exists).
• Understand the connection between the definite integral and area under a function, the fact that continuous functions are integrable, and that not every function is integrable on a given interval.
• Know the properties of definite and indefinite integrals and be able to apply them.
• Be able to find the average value of a function on a given interval.
• **Memorize** both parts of the statement of the fundamental theorem of calculus.
• Be able to apply the fundamental theorem of calculus to evaluate definite integrals of continuous functions and to do computations involving both integration and differentiation.
• Understand the relationship between differentiation and definite and indefinite integrals.
• Be able to use definite integrals to find the area between to functions on an interval or the area of a region enclosed by two or more functions.
• Know how to estimate a definite integral using the Trapezoid Rule and using the Simpson's Rule.
• Know how to estimate the error for approximations of definite integrals when using the Trapezoid Rule and Simpson's Rule.