Math 261 - Lab 11 **The Chain Rule and Implicit Differentiation** **Name:** _____

Show all work for credit. Also, give exact answers unless otherwise noted.

1. Find the derivative of each of the following functions. Simplify your answers completely.

(a) $f(x) = \sqrt{x^2 + 1}$

(b) $f(x) = \sin\left(\sqrt{x^2 + 1}\right)$

(c) $f(\theta) = \dfrac{2\tan\theta}{(5\theta + 1)^3}$

(d) $f(x) = \tan(x^3)$

(e) $f(x) = \tan^3(x)$

(f) $f(x) = \tan^3(x^3)$

(g) $y = (3x - 7)^3(5x^2 - 3x + 2)^5$

(h) $f(x) = \sec(3x)\sin(3x)$

(i) $f(x) = 3\cos(\cot x)$

(j) $f(x) = \dfrac{2x\cos(x^2)}{\sin 3x}$

2. Find the following higher order derivatives. Simplify your answers completely.

(a) Find $f''(x)$ if $f(x) = \left(x^3 - 1\right)^3$

(b) Find $f''(x)$ if $f(x) = \cos(3x)\cot(x)$

(c) Find $f'''(x)$ if $f(x) = \dfrac{4x - 3}{x + 1}$

(d) Find $f^{(5)}(x)$ if $f(x) = \sin 2x$

3. Assuming that each equation determines a differentiable function f such that $y = f(x)$, find $f'(x)$.

(a) $y^2 + x^2 = 2x + 3y^2$

(b) $2xy = x^2 - \sqrt{y}$

(c) $x \sin y + y \sin x = 1$

(d) $x^2(x - y)^2 = x^2 - y^2$

(e) Find y'' in part (a)

(f) Find y'' in part (c)

4. Find an equation for the tangent line to the graph of $\left(x^2 + y^2\right)^2 = 50xy$ at the point $(2, 4)$.

5. Assume $t^2v^3 = 1$ determines a function $v = f(t)$. Use implicit differentiation to show that $v''(t) = \frac{10}{9}v^4$.