Math 261 - Lab 14 **The Mean Value Theorem** **Name:** ______________________________

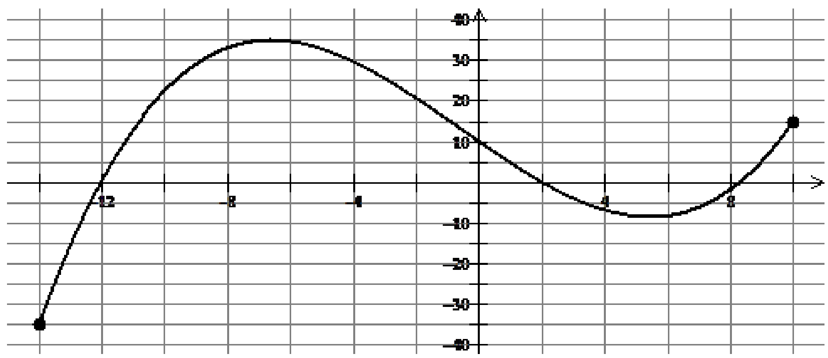
Show all work for credit.

1. Consider the following graph of a function f:

   (a) make a sketch illustrating the conclusion of the Mean Value Theorem.

   (b) Determine the average rate of change

   (c) Estimate all values c that satisfy the conclusion of the MVT.



2. Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

   (a) $s(t) = 3t^2 - 2t + 15$ on $[0, 5]$

   (b) $a(\varphi) = \sin\varphi$ on $\left[0, \frac{3\pi}{2}\right]$

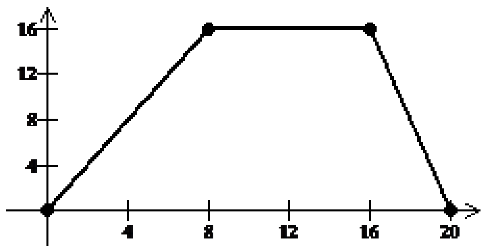
3. Assume that the functions f and g are differentiable for all real numbers, and that g is strictly increasing. The table below gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

   (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

   (b) Explain why there must be a value k for $1 < k < 3$ such that $h'(k) = -5$.

4. The graph below models the velocity (in meters per second) from 0 to 20 seconds of a car on a straight road between two stop signs. Label all solutions with appropriate units.



(a) Find the acceleration at each of the following times.

(i) 8 seconds

(ii) 18 seconds

(b) Write a piece-wise defined function for the acceleration.

(c) Find the average rate of change of the velocity over the time interval from 10 seconds to 18 seconds.

(d) Does the Mean Value Theorem guarantee a value c for $8 < c < 20$? Either find all values c satisfying the Mean Value Theorem on $[8, 20]$ or explain why no such c exists.

5. A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. Recall that the acceleration is the derivative of the velocity with respect to time. The table below shows selected values of these functions.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-20	-30	-20	-14	-10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

(a) For $0 < t < 60$, must there be a time when $v(t) = -5$? Justify your answer.

(b) For $0 < t < 60$, must there be a time when $a(t) = 0$? Justify your answer.