Math 261 - Lab 25 **Review** **Name:** _____

1. (Adapted from the 2005 AP Calculus AB exam). Consider the curve given by $y^2 = 2 + xy$.

   (a) Find $\frac{dy}{dx}$.

   (b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

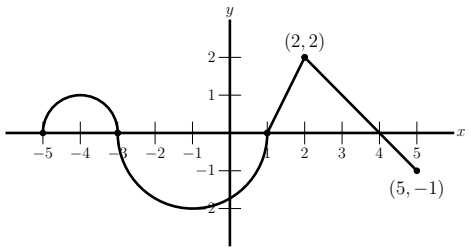
   (c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

2. Find y'' if $y = \dfrac{2-3x}{3x+5}$.

3. An open box is to be made from a 16 inch by 30 inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with largest possible volume?

4. A 6 ft tall woman is walking at a rate of 3 ft/s toward a street light that is 18 feet tall. (a) At what rate is the length of her shadow changing? (b) How fast is the tip of her shadow moving?

5. (From the 2007 AP Calculus AB exam.) Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown below. (This problem is continued on the next page.)



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find

i. $\displaystyle\int_{-5}^{1} f'(x)dx$

ii. $\displaystyle\int_{2}^{5} f'(x)dx$

6. Compute the following (in problems (g) - (i) compute the derivative).

(a) $\lim_{x\to 2} \dfrac{\sqrt{x+2}-2}{x-2}$

(b) $\lim_{x\to -3} \dfrac{x^2-9}{x^2-2x-3}$

(c) $\lim_{x\to 3^-} \dfrac{x+5}{x-3}$

(d) $\lim_{x\to \infty} \dfrac{3x^5-x^2+7}{2x^5-3x^3+x+12}$

(e) $\lim_{x\to -\infty} \dfrac{x^2-3x+1}{x^3+5}$

(f) $\lim_{x\to \infty} \dfrac{-3x^2+5}{x+3}$

(g) $f(x) = 3x^2 - \csc(x) + \tan(x)$

(h) $g(x) = \dfrac{\cos(x)}{1+x^2}$

(i) $h(x) = (x^2-1)\sin(x)$

(j) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}dx$

(k) $\displaystyle\int_1^3 x^3+x^2+x+1dx$

(l) $\displaystyle\int (1+\cos(3x))^5 \sin(3x)dx$