1. Use a δ - ϵ proof to prove that: $\lim\limits_{x \to 3} 2x - 4 = 2.$

2. Use a δ - ϵ proof to prove that: $\lim\limits_{x \to 2} 2 - 5x = -8.$

3. For each of the following, find the limit if it exists.
   If the limit does not exist, write DNE, ∞, or $-\infty$ (whichever is appropriate).

(a) $\lim\limits_{x \to 8} 7$

(b) $\lim\limits_{w \to 3} \dfrac{w^2 - 6w + 9}{w^2 - 9}$

(c) $\lim\limits_{m \to 5} \dfrac{\frac{1}{m} - \frac{1}{5}}{m - 5}$

(d) $\lim\limits_{h \to 0} \dfrac{(x + h)^3 - x^3}{h}$

(e) $\lim\limits_{p \to -7^+} \dfrac{\sqrt{p + 7}}{p + 4}$

(f) $\lim\limits_{t \to 4^+} \left(\dfrac{t - 4}{\sqrt{t^2 - 16}} \right)$

(g) $\lim\limits_{x \to 1^-} \dfrac{|x^2 - 1|}{x^2 - 6x + 5}$

(h) $\lim\limits_{\theta \to 0} \dfrac{\sin^2(\theta)}{4\theta^2}$

(i) $\lim\limits_{x \to 0} \dfrac{3x^2 - 9x + \sin(x)}{2x}$

(j) $\lim\limits_{x \to 1^+} \dfrac{4x}{x^2 - 4x + 3}$

(k) $\lim\limits_{x \to \frac{\pi}{2}^+} \tan x$

(l) $\lim\limits_{x \to 2} f(x)$, where $f(x) =$
$$\begin{cases} -x + 3 & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ x^2 - 3 & \text{if } x > 2 \end{cases}$$

(m) $\lim\limits_{x \to \infty} \dfrac{5x^3 - 7x + 1}{4x^3 - 8}$

(n) $\lim\limits_{x \to \infty} \dfrac{9x^3 - 4x^2 - 5x + 1}{4 - x^2}$

(o) $\lim\limits_{x \to \infty} \dfrac{2x^3 - 4x^2 + 1}{5x^5 - 8x}$

4. For $f(x) = \begin{cases} -x + 3 & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ -x^2 + 6x - 3 & \text{if } x > 2 \end{cases}$ and $g(x) = \begin{cases} \sqrt{x^2 - 2} & \text{if } x < -5 \\ 4x + 5 & \text{if } -5 < x < 5, \\ x^2 & \text{if } x > 5 \end{cases}$

find the following.

(a) $\lim\limits_{x \to 2^+} f(x)$

(e) $\lim\limits_{x \to 5^+} f(x)$

(i) $\lim\limits_{x \to 2^+} g(x)$

(m) $\lim\limits_{x \to 5^+} g(x)$

(b) $\lim\limits_{x \to 2^-} f(x)$

(f) $\lim\limits_{x \to 5^-} f(x)$

(j) $\lim\limits_{x \to 2^-} g(x)$

(n) $\lim\limits_{x \to 5^-} g(x)$

(c) $\lim\limits_{x \to 2} f(x)$

(g) $\lim\limits_{x \to 5} f(x)$

(k) $\lim\limits_{x \to 2} g(x)$

(o) $\lim\limits_{x \to 5} g(x)$
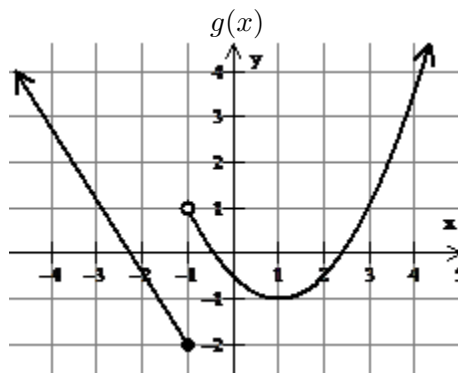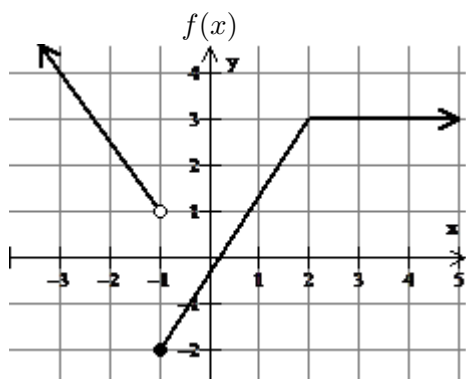
(d) $f(2)$

(h) $f(5)$

(l) $g(2)$

(p) $g(5)$

5. Use the following graphs to determine the value of each of the following.

$f(x)$

$g(x)$

(a) $\displaystyle\lim_{x\to-1^-} g(x) =$

(b) $\displaystyle\lim_{x\to-1^+} g(x) =$

(c) $\displaystyle\lim_{x\to-1} g(x) =$

(d) $g(-1) =$

(e) $\displaystyle\lim_{x\to-1^-} (f+g)(x)$

(f) $\displaystyle\lim_{x\to-1^+} (f+g)(x)$

(g) $\displaystyle\lim_{x\to-1} (f+g)(x)$

(h) $(f+g)(-1)$

(i) $\displaystyle\lim_{x\to1} \frac{f(x)}{g(x)}$

(j) $\left(\dfrac{f}{g}\right)(1)$

(k) $\displaystyle\lim_{x\to-3} (f\circ g)(x)$

(l) $(f\circ g)(-3)$

(m) $\displaystyle\lim_{x\to3} (f\circ g)(x)$

(n) $(f\circ g)(3)$

(o) $(g\circ f)(4)$

(p) $(f\circ g)(4) =$