Math 261

Summation Notation and Area

Summation Notation: Many important mathematical computations require several terms to be added together. Summation notation is used to express the addition process more concisely. That is, if the elements of a set $\{a_1, a_2, \cdots, a_n\}$ are to be added, we represent the sum of this finite collection of elements as follows:

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$$

Examples:

1. $\displaystyle\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 = 15.$
2. $\displaystyle\sum_{k=2}^{4} k^2 - 1 = (2^2 - 1) + (3^2 - 1) + (4^2 - 1) = 3 + 8 + 15 = 26.$
3. $\displaystyle\sum_{k=1}^{7} 3 = 3 + 3 + 3 + 3 + 3 + 3 + 3 = 7(3) = 21.$

Properties of Sums (Theorem 5.11:) Let n be a positive integer and suppose that $\{a_1, a_2, \cdots, a_n\}$ and $\{b_1, b_2, \cdots, b_n\}$ are sets of real numbers. Then:

(I) $\displaystyle\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k.$

(II) $\displaystyle\sum_{k=1}^{n} ca_k = c\left(\sum_{k=1}^{n} a_k\right).$

(III) $\displaystyle\sum_{k=1}^{n}(a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k.$

Special Summation Formulas (Theorem 5.10 & 5.12:)

(I) $\displaystyle\sum_{k=1}^{n} c = nc.$

(II) $\displaystyle\sum_{k=1}^{n} k = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$

(III) $\displaystyle\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$

(IV) $\displaystyle\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$

Examples: