Math 450
Exam 4 Review Sheet

Section 4.6: Adaptive Quadrature

• Understand the idea behind Adaptive Quadrature, including how the error term is approximated by subtracting two approximations using (Compound) Simpson's Rule.
• Be able to carry out a few iterations of Adaptive Quadrature, including checking to see if the error tolerance has been met after an iteration is complete.
• Be able to derive the error estimate formula for Adaptive Quadrature.

Section 4.7: Gaussian Quadrature

• Understand the idea behind Gaussian Quadrature: finding nodes $x_1, x_2, \cdots x_n$ and coefficients $c_1, c_2, \cdots c_n$ such that $\int_{-1}^{1} P(x)\,dx = \sum_{i=1}^{n} c_i P(x_i)$ for any polynomial $P(x)$ of degree less than $2n$.
• Know the key properties of Legendre Polynomials.
• Know the statement and proof of Theorem 4.7
• Given a table of roots and coefficients, be able to use Gaussian Quadrature to approximate the value of definite integrals over $[-1, 1]$.
• Be able to use a change of coordinates to translate an integral over an arbitrary interval $[a, b]$ to one over $[-1, 1]$.

Section 5.1: The Elementary Theory of Initial Value Problems

• Know the definition of a **Lipschitz condition** and a **Lipschitz constant** for a function f over a domain D.
• Know the definition of a **convex** set D.
• Be able to show that a given function f is Lipschitz over a domain D either directly or using Theorem 5.3
• Understand the definition of a **well-posed** initial value problem (including the definition of a **perturbed problem**).
• Be able to use Theorem 5.6 to show that a given initial value problem is well posed.

Section 5.2: Euler's Method

• Understand Euler's method, and be able to derive it from the first order Taylor Polynomial centered at t_i evaluated at t_{i+1}.
• Be able to apply Euler's method to a well posed IVP to approximate the value of the unique solution at a particular point.
• Be able to use the error term from Theorem 5.9 in order to find an upper bound on the error in using Euler's Method to approximate a well posed IVP as a specific point.

Section 5.3: Higher Order Taylor Methods

• Understand the definition of the **local truncation error** of a method for approximating the value of a solution to an IVP.
• Know the general form for both the recursive formula for Taylor's Method of order n and the remainder term that can be used to give an upper bound on the local truncation error.
• Be able to use Taylor's Method of order n to approximate a given well posed IVP at the specific value. This includes knowing how to compute higher order derivatives of $f(t, y)$.

Section 5.4: Runge-Kutta Methods

• Know the statement of Taylor's Theorem for functions of two variables (Theorem 5.13), and the related definitions of Taylor Polynomials in two variables and their Remainder terms.
• Know both the statement of the Midpoint Method and how it was derived by matching coefficients to the two variable Taylor Polynomial of degree one.
• Be able to apply the Midpoint Method, Modified Euler's, Heun's Method, and Runge-Kutta of Order Four to approximate solutions to a well posed IVP at a given point.