Math 450
Numerical Integration Methods

Definition: A *numerical quadrature* method is an approximation of the definite integral that has the following form: $\int_a^b f(x)\,dx = \sum_{i=0}^{n} a_i f(x_i)$. We generally take $a = x_0$ and $b = x_n$.

A. Single Interval Quadrature Formulas

The Trapezoid Rule: $\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right] - \frac{h^3}{12} f''(z)$

Simpson's Rule: $\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] - \frac{h^5}{90} f^{(4)}(z)$

Simpson's 3/8ths Rule: $\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right] - \frac{3h^5}{80} f^{(4)}(z)$

The Midpoint Rule: $\int_{x_{-1}}^{x_1} f(x)\,dx = 2hf(x_0) + \frac{h^3}{3} f''(z)$

Open Newton-Cotes n=1: $\int_{x_{-1}}^{x_2} f(x)\,dx = \frac{3h}{2}\left[f(x_0) + f(x_1)\right] + \frac{3h^3}{4} f''(z)$

Open Newton-Cotes n=2: $\int_{x_{-1}}^{x_3} f(x)\,dx = \frac{4h}{3}\left[2f(x_0) - f(x_1) + 2f(x_2)\right] + \frac{14h^5}{45} f^{(4)}(z)$

Open Newton-Cotes n=3: $\int_{x_{-1}}^{x_4} f(x)\,dx = \frac{5h}{24}\left[11f(x_0) - f(x_1) + f(x_2) + 11f(x_3)\right] + \frac{95h^5}{144} f^{(4)}(z)$

B. Compound Integration Formulas

Composite Trapezoid Rule: $\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right] - \frac{(b-a)h^2}{12} f''(u)$

Composite Midpoint Rule: $\int_a^b f(x)\,dx = 2h\sum_{j=0}^{n/2} f(x_{2j}) + \frac{(b-a)h^2}{6} f''(u)$

Composite Simpson's Rule: $\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{(n/2)} f(x_{2j-1}) + f(b)\right] - \frac{(b-a)h^4}{180} f^{(4)}(u)$

Example: