

Math 450  
Programming Assignment 3  
Due: Tuesday November 25th

1. Suppose that you have been contracted by a local Police Department to write a program that analyzes data gathered by a photo radar system in order to accurately compute the maximum speed of a vehicle based on the data collected. The photo radar system records the initial position of the vehicle, and then computes the total (cumulative) distance traveled from the initial location 0.1 seconds later, 0.2 seconds later, 0.3 seconds later, 0.4 seconds later, and 0.5 seconds later. These distances are recorded in feet.
  - (a) Write a program that takes as input values representing the total distance traveled by a vehicle during the time increments described above and outputs approximations of the instantaneous velocity of the vehicle at the times  $t = 0$ ,  $t = 0.1$ ,  $t = 0.2$ ,  $t = 0.3$ ,  $t = 0.4$ , and  $t = 0.5$ . Make your program as accurate as possible. The contract requires the units of your output to be in miles per hour. [Note: we obtain an extra data point by taking the total distance traveled at time  $t = 0$  to be zero feet].
  - (b) Use your program to determine whether or not the following vehicle who was recorded traveling on a highway with speed limit 65 miles per hour broke the speed limit.

time (in seconds)	total distance (in feet)
0	0
0.1	9.813037037
0.2	17.32607407
0.3	28.57125061
0.4	36.95703703
0.5	44.67407904

2. Write programs that implement each of the following numerical integration methods. Your algorithm should take as input a function  $f(x)$ , an interval  $[a, b]$ , and  $n$ , the number of subdivisions of the interval  $[a, b]$  that will be used.
  - (a) Composite Trapezoid Rule.
  - (b) Composite Simpson's Rule.
  - (c) Romberg Integration [this algorithm may use an additional input  $k$  to represent the final level computed,  $R_{k,k}$ ].
  - (d) Adaptive Quadrature [this algorithm should require an additional input  $\epsilon$ , the error tolerance desired].
3. Use each of your algorithms to approximate  $\int_0^1 \frac{2}{1+x^3} dx$  to within  $\epsilon = 10^{-4}$ . Be sure to make it clear how you know the desired tolerance has been reached.