

Math 450  
Programming Assignment 4  
Due: Friday, December 12th

1. Write programs that implements each of the following numerical methods. Your algorithm should take as input a function  $f(t, y)$ , an interval  $[a, b]$ , an initial condition  $y(a) = \alpha$ , and a step size  $h$ . For the higher order Taylor Methods, you may perform the additional differentiations externally.
  - (a) Euler's Method
  - (b) Taylor's Method of order 2.
  - (c) Taylor's Method of order 4.
  - (d) The Midpoint Method.
  - (e) Modified Euler's Method.
  - (f) Heun's Method.
  - (g) Runge-Kutta Order 4.
  
2. Given the initial value problem  $y' = -ty + 4ty^{-1}$ ,  $0 \leq t \leq 1$ ,  $y(0) = 1$ , use each of your algorithms to approximate the value of  $y(1)$  using each of the algorithms you coded above:
  - (a) For  $h = 0.1$ .
  - (b) For  $h = 0.01$
  
3. Given that the actual solution to this differential equation is:  $y(t) = \sqrt{4 - 3e^{-t^2}}$ 
  - (a) Find the absolute error of each of your estimates using  $h = 0.1$ .
  - (b) Find the absolute error of each of your estimates using  $h = 0.01$ .
  - (c) Compute the ratio of each pair absolute errors you found. Do these ratios give an indication of the rate of convergence of these numerical algorithms?